Integrating out the Standard Higgs Field in the Path Integral

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Abstract

We integrate out the Higgs boson in the electroweak standard model at one loop and construct a low-energy effective Lagrangian assuming that the Higgs mass is much larger than the gauge-boson masses. Instead of applying diagrammatical techniques, we integrate out the Higgs boson directly in the path integral, which turns out to be much simpler. By using the background-field method and the Stueckelberg formalism, we directly find a manifestly gauge-invariant result. The heavy-Higgs effects on fermionic couplings are derived, too. At one loop the $\log M_{\mathrm{H}}$-terms of the heavy-Higgs limit of the electroweak standard model coincide with the UV-divergent terms in the gauged non-linear σ-model, but vertex functions differ in addition by finite constant terms. Finally, the leading Higgs effects to some physical processes are calculated from the effective Lagrangian.

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1 Introduction

In a previous article [1] we have developed a method to eliminate non-decoupling heavy particles from a theory and to construct a one-loop effective Lagrangian which parametrizes the low-energy effects of these heavy particles. We have applied functional methods, i.e. instead of calculating the effects of the the heavy fields diagrammatically, we have integrated them out directly in the path intgral. The contributions of the generated functional determinant to the effective Lagrangian have been expanded in inverse powers of the heavy mass. In Ref. [1] this method has been explained in detail by considering a simple toy model, viz. by integrating out the heavy Higgs boson in an SU(2) gauged linear σ-model without fermions.

In the present article we apply this method to a phenomenologically interesting example: we consider the $\mathrm{SU(2)_W} \times \mathrm{U(1)_Y}$ electroweak standard model (SM) and assume that the Higgs boson has a large mass in comparison to the gauge-boson and fermion masses and the external momenta of the scattering processes under consideration. We integrate out the Higgs boson and determine its non-decoupling effects, i.e. we calculate the $\mathcal{O}(M_\mathrm{H}^0)$-terms (which includes the $\log M_\mathrm{H}$-terms) of the corresponding low-energy effective Lagrangian, including the effective terms with fermion fields. This way we formally construct the limit $M_\mathrm{H} \to \infty$ of the SM at one loop, which is a good approximation to the physically interesting case of a finite but heavy Higgs mass close to the unitarity limit of $M_\mathrm{H} \sim 1$ TeV. The leading one-loop Higgs contributions to scattering processes and physical parameters can then easily be derived from the effective Lagrangian. This will be discussed by considering some examples.

Our method to integrate out heavy fields in the path integral has been discussed in detail in Ref. [1]. Therefore, we will present all those parts of our calculation only very briefly which concern this method in general or which can be done in analogy to the SU(2) model without fermions considered in Ref. [1]. Different methods to construct low-energy effective Lagrangians by integrating out heavy fields have been proposed in [2, 3, 4, 5].

The Higgs boson has recently been integrated out in the SM without fermions by diagrammatic methods in Ref. [6]. The result of our functional calculation agrees with the one given there. Comparing our functional calculation with the diagrammatic one, we find that the functional method simplifies the calculation very much. While in a diagrammatic calculation one has to calculate the Higgs-dependent contributions to various Green functions (i.e. very many Feynman graphs) and then determine the coupling constants of the effective Lagrangian by comparing coefficients ("matching"), in a functional calculation the effective Lagrangian is generated *directly.* For instance, there are 14 effective bosonic interaction terms which are expected to be generated by naive power counting. In fact only 7 of these terms are generated, but the others (viz. the custodial $\mathrm{SU(2)_W}$-violating dimension-4 terms) are not. In a diagrammatic calculation one has first to consider all these terms when comparing the coefficients, and then it turns out that they vanish. However, in a functional calculation it is obvious that they are suppressed by at least a factor $M_\mathrm{W}^2/M_\mathrm{H}^2$. The use of the background-field method [7, 8, 9, 10, 11] and the Stueckelberg Formalism [12, 13, 14, 15] automatically ensures the gauge invariance of the generated effective terms, while in the conventional formalism there are some subleties concerning gauge invariance of the matching conditions [16].

In addition to the treatment of the bosonic sector of the SM, we also determine the effects of a heavy Higgs boson on fermionic interactions, which have not been calculated before. All effective fermionic interactions are proportional to $m_{\mathrm{f}}/M_{\mathrm{W}}$ and thus suppressed for all fermions except for the top quark.

This article is organized as follows: In Sect. 2 we describe the background-field method and the Stueckelberg formalism for the bosonic part of the electroweak standard model and determine the one-loop part of the Lagrangian. In Sect. 3 we diagonalize the Higgs part of this Lagrangian. In Sect. 4 we integrate out the quantum Higgs field and construct the effective Lagrangian, which is written in a manifestly gauge-invariant standard form in Sect. 5. In Sect. 6 we carry out the renormalization of the Higgs sector. In Sect. 7 the background Higgs field is eliminated, which yields the final effective Lagrangian. In Sect. 8 we integrate out the Higgs boson in the fermionic part of the SM and calculate the fermionic terms of the effective Lagrangian. Section 9 contains the discussion of the result. In Sect. 10 we derive the $\log M_{\mathrm{H}}$-contributions to some physical processes directly from our effective Lagrangian. Section 11 contains our conclusions. In App. A the explicit form of the Feynman integrals occurring in the calculations are given. In App. B we prove an identity needed for our calculation.

2 The background-field method and the Stueckelberg formalism

2.1 The standard-model Lagrangian

In this and the subsequent sections we first consider only the bosonic sector of the $\mathrm{SU}(2)_{\mathrm{W}} \times \mathrm{U}(1)_{\mathrm{Y}}$ electroweak SM. The fermions will be included in Sect. 8. The bosonic part of the SM is specified by the Lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\operatorname{tr}\{W_{\mu\nu}W^{\mu\nu}\} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+\frac{1}{2}\operatorname{tr}\left\{(D_\mu\Phi)^\dagger(D^\mu\Phi)\right\} + \frac{1}{2}\mu^2\operatorname{tr}\left\{\Phi^\dagger\Phi\right\} - \frac{1}{16}\lambda\left(\operatorname{tr}\left\{\Phi^\dagger\Phi\right\}\right)^2.\end{aligned} \tag{2.1}$$

The field-strength tensors $W^{\mu\nu}$ and $B^{\mu\nu}$ read

$$\begin{aligned}W^{\mu\nu} &= \partial^\mu W^\nu - \partial^\nu W^\mu - ig_2[W^\mu, W^\nu], \\ B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu,\end{aligned} \tag{2.2}$$

where $W^\mu = W_i^\mu \tau_i/2$ and B^μ represent the corresponding gauge fields. We note that we use the convenient matrix notation for the $\mathrm{SU}(2)_{\mathrm{W}}$ representations throughout, with τ_i denoting the Pauli matrices. The covariant derivative $D^\mu\Phi$ of the scalar Higgs doublet Φ is given by

$$D^\mu\Phi = \partial^\mu\Phi - ig_2 W^\mu\Phi - ig_1\Phi B^\mu\frac{\tau_3}{2}. \tag{2.3}$$

Usually, the field Φ is linearly represented by

$$\Phi = \frac{1}{\sqrt{2}}\left((v+H)\mathbf{1} + 2i\varphi\right), \tag{2.4}$$

where H is the (physical) Higgs field and $\varphi = \varphi_i \tau_i/2$ the (unphysical) Goldstone field. The non-vanishing vacuum expectation value is quantified by

$$v = 2\sqrt{\frac{\mu^2}{\lambda}}. \tag{2.5}$$

For our purpose it is much more appropriate to use the following non-linear representation

$$\Phi = \frac{1}{\sqrt{2}}(v+H)U \qquad \text{with} \quad U = \exp\left(2i\frac{\varphi}{v}\right), \tag{2.6}$$

where H is an $\mathrm{SU(2)_W}$ singlet, and the Goldstone fields φ_i form the unitary matrix U. In both representations the charge eigenstates of φ are given by

$$\varphi^{\pm} = \frac{1}{\sqrt{2}}\left(\varphi_2 \pm i\varphi_1\right), \qquad \chi = -\varphi_3. \tag{2.7}$$

The different representations (2.4) and (2.6) are physically equivalent [13, 15], i.e. both yield the same S-matrix. Inserting (2.6) into the Lagrangian (2.1), one obtains

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\operatorname{tr}\{W_{\mu\nu}W^{\mu\nu}\} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{4}(v+H)^2 \operatorname{tr}\left\{(D_\mu U)^\dagger (D^\mu U)\right\} \\ &+ \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{16}\lambda(v+H)^4. \end{aligned} \tag{2.8}$$

In this form the advantage of the non-linear representation of Φ is apparent. Owing to the unitarity of U the unphysical Goldstone field φ only enters the kinetic term of the scalar fields, but drops out in the cubic and quartic scalar self interactions.

Our conventions and notation for the parameters and fields follow the ones of Refs. [10, 11, 17]. Moreover, substituting $g_2 \to g$, $g_1 \to 0$, $B^\mu \to 0$ reproduces the results of Ref. [1] for the pure SU(2) theory.

Finally, we consider the case of a very heavy Higgs boson, i.e. the limit $M_\mathrm{H} \to \infty$. At tree level, the Lagrangian (2.8) reduces to the one of the *gauged non-linear σ-model* (GNLSM) [18, 19], which follows from (2.8) simply by disregarding the field H. Beyond tree level the situation is much more complicated, as loop corrections associated with virtual Higgs-boson exchange lead to additional (effective) interactions. Our aim is to integrate out the heavy Higgs field at one loop and to construct the corresponding one-loop effective Lagrangian. However, the Lagrangian (2.8) contains the field H up to quartic power so that Gaussian integration is not directly applicable in the path integral. At one loop this problem is circumvented by the background-field method (BFM).

2.2 The background-field method

The BFM [7, 8] was applied to the SM with linearly realized Higgs sector in Refs. [9, 10, 11]. For a pure SU(2) gauge theory we generalized the BFM to the non-linear representation of the scalar sector in Ref. [1]. The same procedure also applies to the $\mathrm{SU(2)_W} \times \mathrm{U(1)_Y}$ SM. Accordingly, we split the fields into background and quantum fields as follows:

$$W^\mu \to \hat{W}^\mu + W^\mu, \quad B^\mu \to \hat{B}^\mu + B^\mu, \quad H \to \hat{H} + H, \quad U \to \hat{U}U, \tag{2.9}$$

where the hats mark background fields. In opposite to the gauge and Higgs fields the matrix U (2.6), which contains the Goldstone field φ, is split multiplicatively. Recall that only the quantum fields are quantized, i.e. they represent variables of integration in the path integral. The background fields act as sources for the generation of vertex functions in the effective action. The background fields correspond to tree lines and the quantum fields to lines in loops. Thus, at one loop only the part of the Lagrangian quadratic in the quantum fields is relevant, and therefore Gaussian integration is applicable. Furthermore, this means that for the construction of vertex functions only the gauge of the quantum fields has to be fixed. Choosing the gauge-fixing term for the quantum fields such that gauge invariance with respect to the background fields is retained, the effective action is "background-gauge-invariant", too. For the linearly realized Higgs sector (2.4) an appropriate gauge-fixing term was given in Refs. [9, 10, 11], for the non-linear case (2.6) we use

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{\xi_W} \operatorname{tr}\left\{\left(\hat{D}^\mu_W W_\mu + \frac{1}{2}\xi_W g_2 v \hat{U}\varphi\hat{U}^\dagger\right)^2\right\} - \frac{1}{2\xi_B}\left(\partial^\mu B_\mu + \frac{1}{2}\xi_B g_1 v \varphi_3\right)^2 \tag{2.10}$$

with

$$\hat{D}^\mu_W X = \partial^\mu X - i g_2 [\hat{W}^\mu, X], \tag{2.11}$$

which is the natural extension of the choice made in Ref. [1] for the SU(2) model. In the following we set $\xi = \xi_W = \xi_B$ in order to avoid mixing between the neutral gauge fields A, Z at tree level. It is straightforward to check that Lagrangian (2.8) with $\mathcal{L}_{\mathrm{gf}}$ of (2.10) leads to an effective action which is invariant under the following background gauge transformation:

$$\hat{W}^\mu \to S\left(\hat{W}^\mu + \frac{i}{g_2}\partial^\mu\right)S^\dagger, \quad \hat{B}^\mu \to \hat{B}^\mu + \partial^\mu\theta_{\mathrm{Y}}, \quad \hat{H} \to \hat{H}, \quad \hat{U} \to S\hat{U}S_{\mathrm{Y}} \tag{2.12}$$

with

$$S = \exp\left(i g_2 \theta\right), \qquad S_{\mathrm{Y}} = \exp\left(i g_1 \theta_{\mathrm{Y}} \frac{\tau_3}{2}\right), \tag{2.13}$$

associated with the following substitution of the quantum fields in the path integral:

$$W^\mu \to S W^\mu S^\dagger, \quad B^\mu \to B^\mu, \quad H \to H, \quad U \to S^\dagger_{\mathrm{Y}} U S_{\mathrm{Y}}. \tag{2.14}$$

$\theta = \theta_i \tau_i/2$ and θ_{Y} denote the group parameters of the $\mathrm{SU(2)_W}$ and $\mathrm{U(1)_Y}$, respectively.

The Faddeev–Popov Lagrangian $\mathcal{L}_{\mathrm{ghost}}$, which corresponds to the gauge-fixing term (2.10), is constructed as usual. In particular, $\mathcal{L}_{\mathrm{ghost}}$ neither involves the quantum nor the background Higgs field.

2.3 The Stueckelberg formalism

The gauge of the background fields has not been specified so far and can be chosen independently from the one of the quantum fields. It is most convenient to choose the *unitary gauge* (U-gauge) for the background fields, where all background Goldstone fields disappear. To this end, we use the Stueckelberg formalism [12, 13, 14, 15], which has been generalized to the BFM in Refs. [1, 5]. We apply the Stueckelberg transformation

$$\hat{W}^\mu \to \hat{U}\hat{W}^\mu\hat{U}^\dagger + \frac{i}{g_2}\hat{U}\partial^\mu\hat{U}^\dagger, \qquad \hat{B}^\mu \to \hat{B}^\mu, \qquad W^\mu \to \hat{U}W^\mu\hat{U}^\dagger, \qquad B^\mu \to B^\mu, \tag{2.15}$$

which transforms the W field-strength and covariant derivative as

$$D^\mu \hat{U} U \to \hat{U} D^\mu U, \qquad (\hat{W}^{\mu\nu} + W^{\mu\nu}) \to \hat{U}(\hat{W}^{\mu\nu} + W^{\mu\nu})\hat{U}^\dagger. \tag{2.16}$$

The effect of this transformation on the Lagrangian is to map the matrix $\hat{U}$ to the unit matrix ($\hat{U} \to \mathbf{1}$), but leaving everything else unaffected. The fact that no background Goldstone fields are present in intermediate steps of the heavy-Higgs expansion simplifies our calculation drastically. Inverting the Stueckelberg transformation (2.15) at the end, we recover the result for an arbitrary background gauge.

3 Diagonalizing the Higgs part of the one-loop Lagrangian

As pointed out above, at one loop only those terms of the Lagrangian are relevant which are bilinear in the quantum fields. In the background U-gauge the full one-loop Lagrangian reads

$$\begin{aligned}
\mathcal{L}^{1-\mathrm{loop}} = \; & \mathrm{tr}\left\{W_\mu \left(g^{\mu\nu}\hat{D}_W^2 + \frac{1-\xi}{\xi}\hat{D}_W^\mu \hat{D}_W^\nu + 2ig_2\hat{W}^{\mu\nu}\right) W_\nu\right\} \\
& + \frac{1}{2}B_\mu \left(g^{\mu\nu}\partial^2 + \frac{1-\xi}{\xi}\partial^\mu\partial^\nu\right) B_\nu + \frac{1}{4}g_2^2(v+\hat{H})^2\,\mathrm{tr}\left\{C_\mu C^\mu\right\} \\
& - \mathrm{tr}\left\{\varphi\left(\frac{1}{v^2}\hat{D}_\mu(v+\hat{H})^2\hat{D}^\mu + \frac{1}{4}\xi g_2^2 v^2 + g_2^2\frac{1}{v^2}(v+\hat{H})^2\hat{C}_\mu\hat{C}^\mu\right)\varphi\right\} - \frac{1}{8}\xi g_1^2 v^2\varphi_3^2 \\
& - \frac{1}{2}H\left(\partial^2 - \mu^2 + \frac{3}{4}\lambda(v+\hat{H})^2 - \frac{1}{2}g_2^2\,\mathrm{tr}\left\{\hat{C}_\mu\hat{C}^\mu\right\}\right)H \\
& - 2g_2\frac{1}{v}(v+\hat{H})H\,\mathrm{tr}\left\{\hat{C}_\mu\partial^\mu\varphi\right\} - 2ig_1g_2\frac{1}{v}(v+\hat{H})H\,\mathrm{tr}\left\{\varphi\hat{W}_\mu\tau_3\right\}\hat{B}^\mu \\
& + g_2^2(v+\hat{H})H\,\mathrm{tr}\left\{\hat{C}_\mu C^\mu\right\} - g_2\frac{1}{v}(2v+\hat{H})\hat{H}\,\mathrm{tr}\left\{C_\mu\partial^\mu\varphi\right\} \\
& - 2ig_2^2 v\,\mathrm{tr}\left\{W_\mu\hat{W}^\mu\varphi\right\} + ig_1g_2\frac{1}{v}(v+\hat{H})^2\left(B_\mu\,\mathrm{tr}\left\{\tau_3\hat{W}^\mu\varphi\right\} + \hat{B}_\mu\,\mathrm{tr}\left\{\tau_3 W^\mu\varphi\right\}\right) \\
& + \mathcal{L}_{\mathrm{ghost}}.
\end{aligned} \tag{3.1}$$

The auxiliary background field $\hat{C}^\mu$ occurring in (3.1) is defined via

$$\hat{C}^\mu = \hat{W}^\mu + \frac{g_1}{g_2}\hat{B}^\mu\frac{\tau_3}{2} = \frac{1}{2}\left(\hat{W}_1^\mu\tau_1 + \hat{W}_2^\mu\tau_2 + \frac{1}{c_{\mathrm{W}}}\hat{Z}^\mu\tau_3\right) \tag{3.2}$$

and the corresponding quantum field analogously.

Since the ghost Lagrangian $\mathcal{L}_{\mathrm{ghost}}$ is bilinear in the Faddeev-Popov ghost fields, which do not have a background part, the one-loop part of $\mathcal{L}_{\mathrm{ghost}}$ in (3.1) contains no other quantum fields than ghosts and remains unaffected by all following manipulations.

Fortunately, not all terms of $\mathcal{L}^{1-\mathrm{loop}}$ in (3.1) are relevant for the construction of the effective Lagrangian describing the non-decoupling effects. In the following we only consider contributions of $\mathcal{O}(M_{\mathrm{H}}^0)$, i.e. we neglect all terms which yield no effects in the limit

$M_{\rm H} \to \infty$. Our complete method for the $1/M_{\rm H}$-expansion was described in detail in Ref. [1] for the SU(2) case. Thus, here we shorten the presentation to the most important steps and omit more technical details. We write the one-loop Lagrangian in the symbolic form

$$\begin{aligned}
\mathcal{L}^{1-\rm loop} = &-\frac{1}{2} H\, \Delta_{H}\, H + H\ \mathrm{tr}\left\{X^{\mu}_{H\overline{W}}\, \overline{W}_{\mu}\right\} + H\ \mathrm{tr}\left\{X_{H\varphi}\, \varphi\right\} \\
&+\ \mathrm{tr}\left\{\overline{W}_{\mu}\, \Delta^{\mu\nu}_{\overline{W}}\, \overline{W}_{\nu}\right\} + \frac{1}{2}\,\mathrm{tr}\left\{A_{\mu}\, \Delta^{\mu\nu}_{A}\, A_{\nu}\right\} +\ \mathrm{tr}\left\{A_{\mu}\, X^{\mu\nu}_{A\overline{W}}\, \overline{W}_{\nu}\right\} \\
&-\ \mathrm{tr}\left\{\varphi\, \Delta_{\varphi}\, \varphi\right\} +\ \mathrm{tr}\left\{\overline{W}_{\mu}\, X^{\mu}_{\overline{W}\varphi}\, \varphi\right\} +\ \mathrm{tr}\left\{A_{\mu}\, X^{\mu}_{A\varphi}\, \varphi\right\} \quad + \mathcal{L}_{\rm ghost}
\end{aligned} \tag{3.3}$$

with the modified quantum $\mathrm{SU}(2)_{\rm W}$ field

$$\overline{W}^{\mu} = \frac{1}{2}\left(W^{\mu}_{1}\tau_{1} + W^{\mu}_{2}\tau_{2} + Z^{\mu}\tau_{3}\right) \tag{3.4}$$

and the quantum photon field A^{μ}. Obviously, there is no AH-term in (3.1).

Applying Gaussian integration over H in the path integral directly to $\mathcal{L}^{1-\rm loop}$ of (3.3), the terms linear in the quantum Higgs field H would yield (problematic) terms with inverse operators acting on quantum fields. However, the terms linear in H can be removed by appropriate shifts of the quantum fields [1, 2, 5]. Substituting successively [1]

$$\begin{aligned}
\varphi &\to \varphi + \frac{1}{2}\hat{\Delta}^{-1}_{\varphi} X^{\dagger}_{H\varphi}\, H + \frac{1}{2}\hat{\Delta}^{-1}_{\varphi} X^{\mu\dagger}_{\overline{W}\varphi}\, \overline{W}_{\mu}, \\
\overline{W}^{\mu} &\to \overline{W}^{\mu} - \frac{1}{2}\hat{\tilde{\Delta}}^{-1\mu\nu}_{\overline{W}}\, \tilde{X}^{\dagger}_{H\overline{W},\nu} H, \\
\varphi &\to \varphi - \frac{1}{2}\hat{\Delta}^{-1}_{\varphi} X^{\mu\dagger}_{\overline{W}\varphi}\, \overline{W}_{\mu}
\end{aligned} \tag{3.5}$$

with

$$\begin{aligned}
\tilde{\Delta}^{\mu\nu}_{\overline{W}} &= \Delta^{\mu\nu}_{\overline{W}} + \frac{1}{4} X^{\mu}_{\overline{W}\varphi} \hat{\Delta}^{-1}_{\varphi} X^{\nu\dagger}_{\overline{W}\varphi}, \\
\tilde{X}_{H\overline{W},\mu} &= X_{H\overline{W},\mu} + \frac{1}{2} X_{H\varphi} \hat{\Delta}^{-1}_{\varphi} X^{\dagger}_{\overline{W}\varphi,\mu}
\end{aligned} \tag{3.6}$$

completely eliminates the $H\overline{W}$- and $H\varphi$-terms without changing the $\overline{W}\varphi$-mixing. The bilinear H-operator transforms into

$$\Delta_{H} \;\to\; \tilde{\Delta}_{H} = \Delta_{H} - \frac{1}{2}\,\mathrm{tr}\left\{X_{H\varphi}\hat{\Delta}^{-1}_{\varphi} X^{\dagger}_{H\varphi}\right\} + \frac{1}{2}\,\mathrm{tr}\left\{\tilde{X}_{H\overline{W},\mu}\hat{\tilde{\Delta}}^{-1\mu\nu}_{\overline{W}}\, \tilde{X}^{\dagger}_{H\overline{W},\nu}\right\}. \tag{3.7}$$

The meaning of the hats over the inverse operators will be explained below. In contrast to the SU(2) case, the transformations (3.5) produce mixing terms between the quantum Higgs field H and the photon field A. Analogously to (3.5), these AH-terms can also be removed by suitable (but more involved) shifts without affecting the H-independent contributions. Only $\tilde{\Delta}_{H}$ is modified again. However, these additional terms in $\tilde{\Delta}_{H}$ only yield $\mathcal{O}(M_{\rm H}^{-2})$-contributions in the subsequent $1/M_{\rm H}$-expansion, and thus are not explicitly discussed here. This can easily be seen as follows: In Ref. [1] it has been shown that

the Yang-Mills couplings and the vector-Goldstone term yield no $\mathcal{O}(M_{\rm H}^0)$-contributions when integrating out the Higgs field and can thus be neglected. However, the quantum photon field A only couples to the other quantum fields through the Yang-Mills and the vector-Goldstone term. Thus, at $\mathcal{O}(M_{\rm H}^0)$ this field may be dropped in (3.3) from the beginning. At the diagrammatical level this means that there are no $\mathcal{O}(M_{\rm H}^0)$-contributions from loops with both photon and Higgs fields, which is in accordance with the diagrammatical calculation in Ref. [6]. Taking only into account effects of $\mathcal{O}(M_{\rm H}^0)$, $\tilde{\Delta}_H$ reduces to

$$\tilde{\Delta}_H \to \tilde{\tilde{\Delta}}_H = \Delta_H - \frac{1}{2}\,\mathrm{tr}\left\{X_{H\varphi}\hat{\Delta}_\varphi^{-1}X_{H\varphi}^\dagger\right\} + \frac{1}{2}\,\mathrm{tr}\left\{X_{H\overline{W},\mu}\hat{\Delta}_{\overline{W},0}^{-1\mu\nu}X_{H\overline{W},\nu}^\dagger\right\} \tag{3.8}$$

as in Ref. [1]. In (3.8) we already made use of the fact that only the lowest-order part $\Delta_{\overline{W},0}^{\mu\nu}$ of $\Delta_{\overline{W}}^{\mu\nu}$ contributes in $\mathcal{O}(M_{\rm H}^0)$, in analogy to the situation in the SU(2) case.

We still have to supply the meaning of the hat over the inverse operators in the previous formulas. As in Ref. [1], $\hat{\Delta}^{-1}$ denotes the restriction of the hermitian, 2×2-matrix-valued inverse operator Δ^{-1} to the subspace spanned by the Pauli matrices τ_i. Only with this restriction the shifts (3.5) make sense, because it ensures that the rhs of these shifts are linear combinations of the Pauli matrices [1]. In terms of a perturbative expansion $\hat{\Delta}^{-1}$ is given by

$$\begin{aligned}\hat{\Delta}^{-1} &= \Delta_0^{-1}P\,\sum_{n=0}^{\infty}(-\Pi\Delta_0^{-1}P)^n \\ &= \Delta_0^{-1}P \;-\; \Delta_0^{-1}P\Pi\Delta_0^{-1}P \;+\; \Delta_0^{-1}P\Pi\Delta_0^{-1}P\Pi\Delta_0^{-1}P \;-\; \cdots,\end{aligned} \tag{3.9}$$

where Δ_0 denotes the lowest-order contribution (which is proportional to the unit matrix) to the full operator $\Delta = \Delta_0 + \Pi$. The operator P is the projector onto the subspace spanned by the τ_i. More generally, we define

$$\begin{aligned}P_i X &= \frac{1}{2}\tau_i\,\mathrm{tr}\left\{\tau_i X\right\} \qquad \text{(no summation over } i\text{)},\\ P &= \sum_{i=1}^{3}P_i,\end{aligned} \tag{3.10}$$

where the P_i project on the single Pauli matrices τ_i, respectively.

For the operators Δ, X of the one-loop Lagrangian (3.3) we just give the terms which are relevant for $\tilde{\tilde{\Delta}}_H$ in (3.8), namely

$$\begin{aligned}\Delta_{\overline{W},0}^{\mu\nu} &= g^{\mu\nu}\partial^2 + \frac{1-\xi}{\xi}\partial^\mu\partial^\nu + g^{\mu\nu}M_{\rm W}^2\left(1+\frac{s_{\rm W}^2}{c_{\rm W}^2}P_3\right),\\ \Delta_\varphi &= \hat{D}^\mu\left(1+\frac{\hat{H}}{v}\right)^2\hat{D}_\mu + g_2^2\hat{C}^\mu\hat{C}_\mu\left(1+\frac{\hat{H}}{v}\right)^2 + \xi M_{\rm W}^2\left(1+\frac{s_{\rm W}^2}{c_{\rm W}^2}P_3\right),\\ \Delta_H &= \partial^2 + M_{\rm H}^2 + \frac{3}{2}M_{\rm H}^2\frac{\hat{H}}{v}\left(2+\frac{\hat{H}}{v}\right) - \frac{1}{2}g_2^2\,\mathrm{tr}\left\{\hat{C}^\mu\hat{C}_\mu\right\},\\ X_{H\overline{W}}^\mu &= 2g_2\left(1+\frac{\hat{H}}{v}\right)M_{\rm W}\hat{C}^\mu\left(1+\frac{1-c_{\rm W}}{c_{\rm W}}P_3\right),\\ X_{H\varphi} &= 2g_2\left(1+\frac{\hat{H}}{v}\right)\left(-\hat{C}^\mu\partial_\mu + ig_1\hat{B}^\mu\tau_3\hat{W}_\mu\right).\end{aligned} \tag{3.11}$$

After all these manipulations the resulting one-loop Lagrangian is obtained from (3.3) upon disregarding $X^{\mu}_{H\overline{W}}$, $X_{H\varphi}$ and replacing Δ_H by $\tilde{\tilde{\Delta}}_H$ of (3.8), where terms yielding only $\mathcal{O}(M_{\rm H}^{-2})$-contributions are neglected.

4 Integrating out the quantum Higgs field and $1/M_{\rm H}$-expansion

The next step is to perform the path integral over the quantum field H by Gaussian integration. For a detailed discussen of this procedure, we again refer to Ref. [1]. The term quadratic in H yields a functional determinant which can be expressed in terms of an effective Lagrangian [1, 4]

$$\mathcal{L}_{\rm eff} = \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log\left(\tilde{\tilde{\Delta}}_H(x, \partial_x + ip)\right). \tag{4.1}$$

$\tilde{\tilde{\Delta}}_H(x, \partial_x + ip)$ can be expanded in terms of derivatives[1],

$$\begin{aligned}\tilde{\tilde{\Delta}}_H(x, \partial_x + ip) &= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \left[\frac{\partial^n}{\partial p_{\mu_1} \dots \partial p_{\mu_n}} \tilde{\tilde{\Delta}}_H(x, ip)\right] \partial_{\mu_1} \dots \partial_{\mu_n} \\ &= -p^2 + M_{\rm H}^2 + \Pi(x, p, \partial_x),\end{aligned} \tag{4.2}$$

leading to the following expansion of the logarithm

$$\log \tilde{\tilde{\Delta}}_H(x, \partial_x + ip) = \log(-p^2 + M_{\rm H}^2) - \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\Pi}{p^2 - M_{\rm H}^2}\right)^n. \tag{4.3}$$

The first log-term of (4.3) yields a constant contribution to the effective Lagrangian, which is irrelevant in this context and will be dropped in the following. The powers of Π in (4.3) contain propagator terms $(p^2 - M^2)^{-m}$ with $M^2 = M_{\rm W}^2, M_{\rm Z}^2, \xi M_{\rm W}^2$ or $\xi M_{\rm Z}^2$ originating from the derivative expansion of the inverse propagators $\hat{\Delta}_{\varphi}^{-1}$, $\hat{\Delta}_{\overline{W},0}^{-1\mu\nu}$. Hence, upon inserting expansion (4.3) into (4.1), the effective Lagrangian can be expressed in terms of one-loop vacuum integrals of the type

$$I^i_{klm}(\xi)\, g_{\mu_1 \dots \mu_{2k}} = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^Dp \frac{p_{\mu_1} \dots p_{\mu_{2k}}}{(p^2 - M_{\rm H}^2)^l (p^2 - \xi M_i^2)^m}, \qquad M_i = M_{\rm W}, M_{\rm Z}. \tag{4.4}$$

In (4.4) it is already indicated that we use dimensional regularization throughout with D denoting the number of space-time dimensions, and μ representing the reference mass scale. $g_{\mu_1 \dots \mu_{2k}}$ is the totally symmetric tensor of rank $2k$ built of the metric tensor $g_{\mu\nu}$. For $D \to 4$ the integrals $I^i_{klm}(\xi)$ are $\mathcal{O}(M_{\rm H}^n)$ with

$$n = 4 + 2(k - l - m) \tag{4.5}$$

[1]The first line of (4.2) cannot be taken literally for the derivative expansion. The partial derivatives do not commute with the background fields in $\tilde{\tilde{\Delta}}_H(x, ip)$, and thus one also has to take care of the position of the derivative operators, which can easily be achieved in the actual calculation.

if $n \geq 0$, and $\mathcal{O}(M_{\rm H}^{-2})$ or less if $n < 0$. The explicit expressions for the integrals relevant for $\mathcal{L}_{\rm eff}$ are listed in App. A. In particular, the $\mathcal{O}(M_{\rm H}^0)$-parts of all logarithmically divergent integrals are independent of ξ and M_i^2. Consequently, the index i and the argument ξ will be dropped for these in the following. In addition to the $M_{\rm H}$-dependence of the integrals, there is an explicit $M_{\rm H}$-dependence in the generated effective Lagrangian due to the Higgs self interactions and an implicit $M_{\rm H}$-dependence stemming from the occurrence of the background Higgs field $\hat{H}$ which will later be eliminated by a propagator expansion yielding $\hat{H} = \mathcal{O}(M_{\rm H}^{-2})$. Thus, as in Ref. [1], we introduce an auxiliary power-counting parameter ζ, which counts the powers of p_μ, $\hat{H}$ and $M_{\rm H}$ according to

$$p_\mu \to \zeta, \qquad M_{\rm H} \to \zeta, \qquad \hat{H} \to \zeta^{-2}. \tag{4.6}$$

In order to obtain the effective Lagrangian at $\mathcal{O}(M_{\rm H}^0)$, we only have to consider contributions up to $\mathcal{O}(\zeta^{-4})$ in the expansion of $\log \tilde{\tilde{\Delta}}_H(x, \partial_x + ip)$ (i.e. up to $\mathcal{O}(\zeta^{-2})$ in $\tilde{\tilde{\Delta}}_H(x, \partial_x + ip)$) and can neglect higher negative powers of ζ.

As a result of this power counting it turns out that most of the contributions of the projection operator P_3 (3.10) in $\Delta^{\mu\nu}_{W,0}$ and Δ_φ (3.11) can be neglected at $\mathcal{O}(M_{\rm H}^0)$. In order to illustrate this, we consider the operator $\hat{\Delta}_\varphi^{-1}(x, \partial_x + ip)$, which occurs in (4.1) with (3.8). Using (3.10) we write

$$M_{\rm W}^2 \left(1 + \frac{s_{\rm W}^2}{c_{\rm W}^2} P_3\right) P = M_i^2 P_i, \qquad \text{with} \qquad M_{1,2} = M_{\rm W}, \quad M_3 = M_Z \tag{4.7}$$

and find with (3.9)

$$\begin{aligned}
\hat{\Delta}_\varphi^{-1}(x, \partial_x + ip) = & -\frac{1}{p^2 - \xi M_i^2} P_i \\
& - \frac{1}{(p^2 - \xi M_i^2)(p^2 - \xi M_j^2)} P_i \left[-\frac{2\hat{H}}{v} p^2 + 2ip_\mu \hat{D}^\mu + \hat{D}^2 + g_2^2 \hat{C}^\mu \hat{C}_\mu \right] P_j \\
& + \frac{1}{(p^2 - \xi M_i^2)(p^2 - \xi M_j^2)(p^2 - \xi M_k^2)} 4P_i (p_\mu \hat{D}^\mu) P_j (p_\nu \hat{D}^\nu) P_k \\
& + \mathcal{O}(\zeta^{-5}).
\end{aligned} \tag{4.8}$$

The operator $(p^2 - \xi M_i^2)^{-1} P_i$ occurring several times in this expression can be written as

$$\frac{1}{p^2 - \xi M_i^2} P_i = \frac{1}{p^2 - \xi M_{\rm W}^2} P - \xi \frac{M_{\rm W}^2 - M_{\rm Z}^2}{(p^2 - \xi M_{\rm W}^2)(p^2 - \xi M_{\rm Z}^2)} P_3. \tag{4.9}$$

The second term in (4.9) is $\mathcal{O}(\zeta^{-4})$ and can thus be neglected in the second and the third term of (4.8), because $\hat{\Delta}_\varphi^{-1}(x, \partial_x + ip)$ is only needed at $\mathcal{O}(\zeta^{-4})$.

Expanding $\log \tilde{\tilde{\Delta}}_H(x, \partial_x + ip)$ and integrating over p in analogy to Ref. [1], we find the effective Lagrangian according to (4.1), (4.2) and (4.3):

$$\mathcal{L}_{\rm eff} = \frac{1}{16\pi^2} \Bigg\{ \quad I_{010} \left[\frac{3 g_2 M_{\rm H}^2}{4 M_{\rm W}} \hat{H} + \frac{3 g_2^2 M_{\rm H}^2}{16 M_{\rm W}^2} \hat{H}^2 - \frac{1}{4} g_2^2 \,\mathrm{tr}\left\{ \hat{C}_\mu \hat{C}^\mu \right\} \right]$$

$$
\begin{aligned}
&- I_{011}\; g_2^2 M_i^2 \,\mathrm{tr}\left\{\hat{C}_\mu P_i \hat{C}^\mu\right\} \\
&+ I_{111}^i(1)\; g_2^2 \,\mathrm{tr}\left\{\hat{C}_\mu P_i \hat{C}^\mu\right\} \\
&+ I_{011} g_2^2 \Big[\mathrm{tr}\left\{(\partial_\mu \hat{C}^\mu)^2\right\} + 2ig_1 \hat{B}_\mu \,\mathrm{tr}\left\{\tau_3 \hat{W}^\mu (\partial_\nu \hat{C}^\nu)\right\} \\
&\qquad\qquad + g_1^2 \hat{B}_\mu \hat{B}_\nu \,\mathrm{tr}\left\{\tau_3 \hat{W}^\mu P \hat{W}^\nu \tau_3\right\}\Big] \\
&+ I_{112} g_2^2 \Big[-4\,\mathrm{tr}\left\{(\partial_\mu \hat{C}^\mu)(\hat{D}_\nu \hat{C}^\nu)\right\} + \mathrm{tr}\left\{\hat{C}_\mu \hat{D}^2 \hat{C}^\mu\right\} \\
&\qquad\qquad + g_2^2 \,\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu \hat{C}_\nu \hat{C}^\nu\right\} - 4ig_1 \hat{B}_\mu \,\mathrm{tr}\left\{\tau_3 \hat{W}^\mu P \hat{D}_\nu \hat{C}^\nu\right\}\Big] \\
&- I_{213} 4 g_2^2 \Big[\mathrm{tr}\left\{\hat{C}_\mu \hat{D}^\mu P \hat{D}^\nu \hat{C}_\nu\right\} + \mathrm{tr}\left\{\hat{C}_\mu \hat{D}^\nu P \hat{D}^\mu \hat{C}_\nu\right\} + \mathrm{tr}\left\{\hat{C}_\mu \hat{D}_\nu P \hat{D}^\nu \hat{C}^\mu\right\}\Big] \\
&+ I_{020} \left[\frac{9 g_2^2 M_\mathrm{H}^4}{16 M_\mathrm{W}^2} \hat{H}^2 - \frac{3 g_2^3 M_\mathrm{H}^2}{8 M_\mathrm{W}} \hat{H} \,\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\} + \frac{1}{16} g_2^4 \left(\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\}\right)^2 \right] \\
&+ I_{121} \left[\frac{3 g_2^3 M_\mathrm{H}^2}{2 M_\mathrm{W}} \hat{H} \,\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\} - \frac{1}{2} g_2^4 \left(\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\}\right)^2 \right] \\
&+ I_{222} g_2^4 \left[\left(\mathrm{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\}\right)^2 + 2 \left(\mathrm{tr}\left\{\hat{C}_\mu \hat{C}_\nu\right\}\right)^2 \right] \Bigg\} \\
+\,& \mathcal{O}(\zeta^{-2}), \qquad\qquad (4.10)
\end{aligned}
$$

where we have used the notation (4.4) for the (vacuum) one-loop integrals.

The origin of the various terms in (4.10) is the following: The first line is the contribution of Δ_H in (3.8), the second stems from of $X_{H\overline{W},\mu} \hat{\Delta}^{-1\mu\nu}_{\overline{W},0} X^\dagger_{H\overline{W},\nu}$, the third gets contributions from $X_{H\varphi} \hat{\Delta}_\varphi^{-1} X^\dagger_{H\varphi}$ and $X_{H\overline{W},\mu} \hat{\Delta}^{-1\mu\nu}_{\overline{W},0} X^\dagger_{H\overline{W},\nu}$ together, and the remaining terms come from $X_{H\varphi} \hat{\Delta}_\varphi^{-1} X^\dagger_{H\varphi}$.

5 Introducing standard traces and inverting the Stueckelberg transformation

The effective Lagrangian (4.10) has to be written in a more convenient form. Since we want to invert the Stueckelberg transformation (2.15) in order to obtain $\mathcal{L}_\mathrm{eff}$ in an arbitrary background gauge, it is useful to introduce appropriate gauge-invariant standard traces. Such traces have for instance been introduced in Ref. [19][2]. Since we presently work in the U-gauge for the background fields, we express these terms both in their gauge-invariant form (lhs of the arrow) and in the U-gauge (rhs of the arrow):

$$
\begin{aligned}
\mathcal{L}_0 &= M_\mathrm{W}^2 \left(\mathrm{tr}\left\{\hat{T} \hat{V}_\mu\right\}\right)^2 && \stackrel{\mathrm{U-gauge}}{\longrightarrow} && -g_2^2 M_\mathrm{W}^2 \left(\mathrm{tr}\left\{\tau_3 \hat{C}_\mu\right\}\right)^2, \\
\mathcal{L}_1 &= \frac{1}{2} g_2^2 \hat{B}_{\mu\nu} \,\mathrm{tr}\left\{\hat{T} \hat{W}^{\mu\nu}\right\} && \stackrel{\mathrm{U-gauge}}{\longrightarrow} && \frac{1}{2} g_2^2 \hat{B}_{\mu\nu} \,\mathrm{tr}\left\{\tau_3 \hat{W}^{\mu\nu}\right\},
\end{aligned}
$$

[2] In Ref. [19] the couplings constants α_i are part of the effective terms $\mathcal{L}_i$ while here they are not. Apart from this, our terms are identical with those used in Ref. [19]. The $\mathcal{L}_1'$ defined there corresponds to our $\mathcal{L}_0$, and the traces in $\mathcal{L}_6$, …, $\mathcal{L}_{10}$, $\mathcal{L}_{12}$ and $\mathcal{L}_{13}$ of Ref. [19] do not occur in our calculation and thus are not listed here.

$$
\begin{aligned}
\mathcal{L}_2 &= \frac{1}{2} i g_2 \hat{B}_{\mu\nu} \operatorname{tr}\left\{\hat{T}[\hat{V}^\mu, \hat{V}^\nu]\right\} & \stackrel{\text{U-gauge}}{\longrightarrow} &\quad -\frac{1}{2} i g_2^3 \hat{B}_{\mu\nu} \operatorname{tr}\left\{\tau_3[\hat{C}^\mu, \hat{C}^\nu]\right\}, \\
\mathcal{L}_3 &= i g_2 \operatorname{tr}\left\{\hat{W}_{\mu\nu}[\hat{V}^\mu, \hat{V}^\nu]\right\} & \stackrel{\text{U-gauge}}{\longrightarrow} &\quad -i g_2^3 \operatorname{tr}\left\{\hat{W}_{\mu\nu}[\hat{C}^\mu, \hat{C}^\nu]\right\}, \\
\mathcal{L}_4 &= \left(\operatorname{tr}\left\{\hat{V}_\mu \hat{V}_\nu\right\}\right)^2 & \stackrel{\text{U-gauge}}{\longrightarrow} &\quad g_2^4 \left(\operatorname{tr}\left\{\hat{C}_\mu \hat{C}_\nu\right\}\right)^2, \\
\mathcal{L}_5 &= \left(\operatorname{tr}\left\{\hat{V}_\mu \hat{V}^\mu\right\}\right)^2 & \stackrel{\text{U-gauge}}{\longrightarrow} &\quad g_2^4 \left(\operatorname{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\}\right)^2, \\
\mathcal{L}_{11} &= \operatorname{tr}\left\{\left(\hat{D}_W^\mu \hat{V}_\mu\right)^2\right\} & \stackrel{\text{U-gauge}}{\longrightarrow} &\quad -g_2^2 \operatorname{tr}\left\{\left(\hat{D}_W^\mu \hat{C}_\mu\right)^2\right\}
\end{aligned} \tag{5.1}
$$

with $\hat{D}_W$ defined in (2.11). Following Ref. [19], we introduce the shorthand notation

$$
\hat{V}^\mu = \left(\hat{D}^\mu \hat{U}\right) \hat{U}^\dagger, \qquad \hat{T} = \hat{U} \tau_3 \hat{U}^\dagger. \tag{5.2}
$$

First, we consider the terms in (4.10) which contain derivatives or covariant derivatives (2.3). These terms are proportional to I_{011}, I_{112} or I_{213}. We express the derivatives in terms of field-strength tensors (2.2) and vector-covariant derivatives $\hat{D}_W^\mu$ (2.11). These terms become

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}\Big|_{I_{011}}^{\text{deriv}} &= -\frac{1}{16\pi^2} I_{011}\, \mathcal{L}_{11}, \\
\mathcal{L}_{\text{eff}}\Big|_{I_{112}}^{\text{deriv}} &= \frac{1}{16\pi^2} I_{112} \Big[-\frac{1}{2} g_2^2 \operatorname{tr}\left\{\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}\right\} - \frac{1}{4} g_1^2 \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \\
&\qquad - \frac{g_1}{g_2} \mathcal{L}_1 - \frac{1}{2} \frac{g_1}{g_2} \mathcal{L}_2 + \frac{1}{2} \mathcal{L}_3 - \frac{1}{2} \mathcal{L}_5 + 5 \mathcal{L}_{11} \Big], \\
\mathcal{L}_{\text{eff}}\Big|_{I_{213}}^{\text{deriv}} &= \frac{1}{16\pi^2} I_{213} \Big[2 g_2^2 \operatorname{tr}\left\{\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}\right\} + g_1^2 \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \\
&\qquad + 4 \frac{g_1}{g_2} \mathcal{L}_1 + 4 \frac{g_1}{g_2} \mathcal{L}_2 - 4 \mathcal{L}_3 - 4 \mathcal{L}_4 + 4 \mathcal{L}_5 - 12 \mathcal{L}_{11} \Big].
\end{aligned} \tag{5.3}
$$

Next, we consider the terms proportional to I_{011} and $I_{111}^i(1)$ which contain the operators P_i (3.10) with different coefficients for $i = 1, 2$ and $i = 3$. These can easily be evaluated by using

$$
M_i^2 \operatorname{tr}\left\{\hat{C}_\mu P_i \hat{C}^\mu\right\} = M_{\text{W}}^2 \operatorname{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\} + \frac{1}{2} \frac{g_1^2}{g_2^2} M_{\text{W}}^2 \left(\operatorname{tr}\left\{\tau_3 \hat{C}_\mu\right\}\right)^2 \tag{5.4}
$$

and a corresponding identity for $I_{111}^i(1) \operatorname{tr}\left\{\hat{C}_\mu P_i \hat{C}^\mu\right\}$. We find:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}\Big|_{I_{011}}^{P_i} &= \frac{1}{16\pi^2} I_{011} \left[-g_2^2 M_{\text{W}}^2 \operatorname{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\} + \frac{1}{2} \frac{g_1^2}{g_2^2} \mathcal{L}_0 \right], \\
\mathcal{L}_{\text{eff}}\Big|_{I_{111}^i(1)}^{P_i} &= \frac{1}{16\pi^2} \left[I_{111}^W(1)\, g_2^2 \operatorname{tr}\left\{\hat{C}_\mu \hat{C}^\mu\right\} - \left(I_{111}^Z(1) - I_{111}^W(1)\right) \frac{1}{2} \frac{1}{M_{\text{W}}^2} \mathcal{L}_0 \right].
\end{aligned} \tag{5.5}
$$

Finally, we reintroduce the background Goldstone fields $\hat{\varphi}_i$ by inverting the Stueckelberg transformation (2.15), i.e. we transform the background fields $\hat{W}_\mu$ and $\hat{B}_\mu$ as

$$
\hat{W}^\mu \to \hat{U}^\dagger \hat{W}^\mu \hat{U} + \frac{i}{g_2} \hat{U}^\dagger \partial^\mu \hat{U}, \qquad \hat{B}^\mu \to \hat{B}^\mu. \tag{5.6}
$$

The transformations of the fields, field-strength tensors and derivatives in the standard traces (5.1) under the Stueckelberg transformation (5.6) are given by

$$\begin{aligned} \hat{C}^\mu &\to \frac{i}{g_2}\hat{U}^\dagger \hat{V}^\mu \hat{U}, \qquad & \hat{D}_W^\mu \hat{C}_\mu &\to \frac{i}{g_2}\hat{U}^\dagger \left(\hat{D}_W^\mu \hat{V}_\mu\right)\hat{U}, \\ \hat{W}^{\mu\nu} &\to \hat{U}^\dagger \hat{W}^{\mu\nu}\hat{U}, & \hat{B}^{\mu\nu} &\to \hat{B}^{\mu\nu}. \end{aligned} \tag{5.7}$$

Consequently, the traces (5.1) take their gauge-invariant form (lhs of the arrow in (5.1)). Collecting all terms, we find

$$\begin{aligned} \mathcal{L}_{\mathrm{eff}} = \frac{1}{16\pi^2}\Bigg\{ &\quad g_2 \frac{3M_{\mathrm{H}}^2}{4M_{\mathrm{W}}} I_{010}\hat{H} + g_2^2\left(\frac{3M_{\mathrm{H}}^2}{16M_{\mathrm{W}}^2}I_{010} + \frac{9M_{\mathrm{H}}^4}{16M_{\mathrm{W}}^2}I_{020}\right)\hat{H}^2 \\ &+ g_2\left(\frac{3M_{\mathrm{H}}^2}{8M_{\mathrm{W}}}I_{020} - \frac{3M_{\mathrm{H}}^2}{2M_{\mathrm{W}}}I_{121}\right)\hat{H}\,\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\} \\ &+ \left(\frac{1}{4}I_{010} + M_{\mathrm{W}}^2 I_{011} - I_{111}^W(1)\right)\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\} \\ &+ g_2^2\Big(-\frac{1}{2}I_{112} + 2I_{213}\Big)\,\mathrm{tr}\left\{\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right\} + g_1^2\Big(-\frac{1}{4}I_{112} + I_{213}\Big)\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \\ &+ \left(\frac{1}{2}\frac{g_1^2}{g_2^2}I_{011} + \frac{1}{2M_{\mathrm{W}}^2}\left[I_{111}^W(1) - I_{111}^Z(1)\right]\right)\mathcal{L}_0 \\ &+ \frac{g_1}{g_2}\Big(-I_{112} + 4I_{213}\Big)\mathcal{L}_1 + \frac{g_1}{g_2}\Big(-\frac{1}{2}I_{112} + 4I_{213}\Big)\mathcal{L}_2 + \Big(\frac{1}{2}I_{112} - 4I_{213}\Big)\mathcal{L}_3 \\ &+ \Big(-4I_{213} + 2I_{222}\Big)\mathcal{L}_4 + \Big(\frac{1}{16}I_{020} - \frac{1}{2}I_{121} + 4I_{213} + I_{222}\Big)\mathcal{L}_5 \\ &+ \Big(-I_{011} + 5I_{112} - 12I_{213}\Big)\mathcal{L}_{11}\Bigg\} \\ &+ \mathcal{O}(\zeta^{-2}). \end{aligned} \tag{5.8}$$

This Lagrangian is manifestly invariant under the gauge transformations of the background fields (2.12), under which the quantities occurring in (5.8) with (5.1) transform covariantly according to

$$\begin{aligned} &\hat{W}^{\mu\nu} \to S\hat{W}^{\mu\nu}S^\dagger, \qquad \hat{B}^{\mu\nu} \to \hat{B}^{\mu\nu} \\ &\hat{V}^\mu \to S\hat{V}^\mu S^\dagger, \qquad \hat{D}_W^\mu \hat{V}_\mu \to S\left(\hat{D}_W^\mu\hat{V}_\mu\right)S^\dagger, \qquad \hat{T} \to S\hat{T}S^\dagger. \end{aligned} \tag{5.9}$$

The gauge for the background fields can now be fixed arbitrarily.

6 Renormalization

In the previous sections we have dealt with bare parameters and bare fields only. In the following, these bare quantities are marked by a subscript “0”. We apply the renormalization transformation to the parameters

$$\begin{aligned} e &\to e_0 = (1+\delta Z_e)e, \\ M_a^2 &\to M_{a,0}^2 = M_a^2 + \delta M_a^2, \qquad a = W, Z, H, \\ t &\to t_0 = t + \delta t. \end{aligned} \tag{6.1}$$

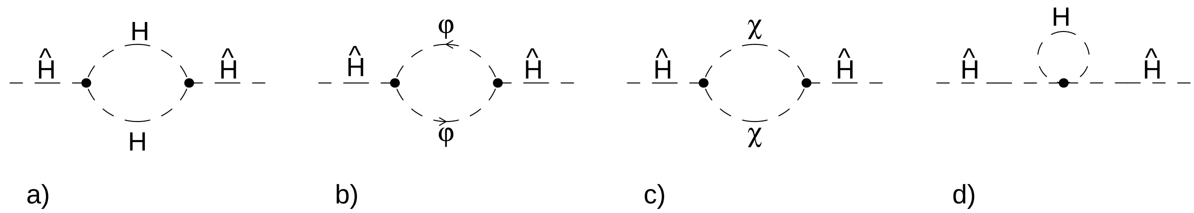


Figure 1: All diagrams of $\mathcal{O}(M_{\rm H}^4)$ contributing to $\delta M_{\rm H}^2$.

The tadpole term $t = v(\mu^2 - \lambda v^2/4)$ is defined in the Lagrangian (2.1) via the term $tH(x)$.

We apply on-shell renormalization [11, 17], where $M_{\rm W}$, $M_{\rm Z}$ and $M_{\rm H}$ represent the physical masses (propagator poles). The electric unit charge is defined in the Thomson limit as usual, and the renormalized tadpole vanishes[3] ($t = 0$). The remaining renormalized parameters are fixed by the relations

$$c_{\rm W} = \frac{M_{\rm W}}{M_{\rm Z}}, \qquad s_{\rm W} = \sqrt{1-c_{\rm W}^2}, \qquad g_1 = \frac{e}{c_{\rm W}}, \qquad g_2 = \frac{e}{s_{\rm W}}, \qquad v = \frac{2M_{\rm W}}{g_2}, \qquad \mu^2 = \frac{M_{\rm H}^2}{2}. \tag{6.2}$$

The on-shell conditions imply for the counterterms in (6.1)

$$\begin{aligned}
\delta M_a^2 &= \mathrm{Re}\left\{\Sigma_{\rm T}^{\hat a\hat a}(M_a^2)\right\}, \qquad a = W, Z, \\
\delta M_{\rm H}^2 &= \mathrm{Re}\left\{\Sigma^{\hat H\hat H}(M_{\rm H}^2)\right\}, \\
\delta Z_e &= \frac{1}{2}\frac{\partial \Sigma_{\rm T}^{\hat A\hat A}(q^2)}{\partial q^2}\bigg|_{q^2=0}, \\
\delta t &= -T^{\hat H},
\end{aligned} \tag{6.3}$$

where $\Sigma_{\rm T}^{\hat A\hat A}$, $\Sigma_{\rm T}^{\hat W\hat W}$, $\Sigma_{\rm T}^{\hat Z\hat Z}$and $\Sigma^{\hat H\hat H}$ represent the transversal parts of the unrenormalized vector-boson self-energies and the unrenormalized $\hat H$–self-energy, respectively[4]. Concerning vertex functions and self-energies our notation follows the one of Refs. [10, 11] throughout. Since δZ_e, $\delta M_{\rm W}^2$, $\delta M_{\rm Z}^2$ and δt are calculated from vertex functions at low-energy scales, i.e. $|q^2| \ll M_{\rm H}^2$, they can be read directly from the effective Lagrangian (5.8), which is constructed at $|q^2| \ll M_{\rm H}^2$. However, $\delta M_{\rm H}^2$ is fixed at $q^2 = M_{\rm H}^2$ and thus cannot be read from (5.8) but has to be calculated diagrammatically. As it turns out below, $\delta M_{\rm H}^2$ is only needed at $\mathcal{O}(M_{\rm H}^4)$ so that we merely have to consider those diagrams contributing to the $\hat H$–self-energy, which have internal Higgs or Goldstone lines but no vector lines, as shown in Fig. 1. We find

[3] This means that the relation (2.5) holds for *renormalized* quantities, whereas for unrenormalized parameters t_0-terms occur. In order to avoid confusion we omitted t in the previous sections, but reintroduce it here.

[4] Note that δZ_e gets no contribution from the $\hat A\hat Z$-mixing self-energy owing to $\Sigma_T^{\hat A\hat Z}(0) = 0$, which follows from the Ward identity $\Sigma_L^{\hat A\hat Z}(q^2) = 0$ [10, 11].

$$
\begin{aligned}
\delta M_{\mathrm{H}}^2 &= \frac{1}{16\pi^2} g_2^2 \frac{3M_{\mathrm{H}}^2}{8M_{\mathrm{W}}^2}\left[M_{\mathrm{H}}^2 \operatorname{Re}\left\{B_0(M_{\mathrm{H}}^2,0,0)\right\} + 3M_{\mathrm{H}}^2 B_0(M_{\mathrm{H}}^2, M_{\mathrm{H}}, M_{\mathrm{H}}) + I_{010}\right] + \mathcal{O}(M_{\mathrm{H}}^2),\\
\delta M_{\mathrm{W}}^2 &= \frac{1}{16\pi^2} g_2^2 \left(\frac{1}{4} I_{010} - I_{111}^W(1)\right) + \mathcal{O}(M_{\mathrm{H}}^0),\\
\delta M_{\mathrm{Z}}^2 &= \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{W}}^2}\delta M_{\mathrm{W}}^2 + \mathcal{O}(M_{\mathrm{H}}^0),\\
\delta t &= -\frac{1}{16\pi^2} g_2 \frac{3M_{\mathrm{H}}^2}{4M_{\mathrm{W}}} I_{010} + \mathcal{O}(M_{\mathrm{H}}^0),\\
\delta Z_e &= \mathcal{O}(M_{\mathrm{H}}^0),
\end{aligned}
\tag{6.4}
$$

where B_0 denotes the general scalar two-point function

$$
B_0(k^2, M_0, M_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D p \, \frac{1}{[p^2 - M_0^2 + i\varepsilon][(p+k)^2 - M_1^2 + i\varepsilon]}. \tag{6.5}
$$

The B_0-terms occurring in (6.4) are explicitly given in App. A.

In addition we introduce the field renormalization

$$
\hat{F} \to \hat{F}_0 = Z_{\hat{F}}^{1/2}\hat{F} = (1 + \frac{1}{2}\delta Z_{\hat{F}})\hat{F}, \qquad F = W, B, H, \varphi. \tag{6.6}
$$

The renormalized Lagrangian remains gauge-invariant [11], if one chooses

$$
\delta Z_{\hat{W}} = -2\frac{\delta g_2}{g_2}, \qquad \delta Z_{\hat{B}} = -2\frac{\delta g_1}{g_1}, \qquad \delta Z_{\hat{\varphi}} = 2\frac{\delta v}{v}, \tag{6.7}
$$

while $\delta Z_{\hat{H}}$ can be chosen arbitrarily. Since $\delta Z_{\hat{H}}$ drops out anyhow when $\hat{H}$ is removed from the theory, we can simply choose

$$
\delta Z_{\hat{H}} = 0. \tag{6.8}
$$

With the choice (6.7) the propagators of the massive gauge bosons acquire residues different from one. However, for the construction of the effective Lagrangian we only need for the gauge-boson field-renormalization constants that $\delta Z_{\hat{W}}$ and $\delta Z_{\hat{B}}$ only get contributions of $\mathcal{O}(M_{\mathrm{H}}^0)$. This means that we could equivalently well normalize the residues of all gauge-boson propagators to one without affecting the final result of the effective Lagrangian. On the other hand, the condition (6.7) for $\delta Z_{\hat{\varphi}}$ is indeed necessary, because it guarantees that the renormalization of the matrix $\hat{U}$ (2.6) does not yield contributions of $\mathcal{O}(M_{\mathrm{H}}^2)$.

As discussed in Ref. [1], we do not have to carry out the complete renormalization for the calculation of the effective Lagrangian. It is sufficient to determine the $\hat{H}$-dependent part of the counterterm Lagrangian

$$
\mathcal{L}_{\hat{\mathrm{H}}}^{\mathrm{ct}} = \delta t \hat{H} - \frac{1}{2}\delta M_{\mathrm{H}}^2 \hat{H}^2 - \frac{1}{2}\frac{\delta M_{\mathrm{W}}^2}{g_2 M_{\mathrm{W}}}\hat{H}\operatorname{tr}\left\{\hat{V}^\mu \hat{V}_\mu\right\} + \mathcal{O}(\zeta^{-2}). \tag{6.9}
$$

This part yields contributions when eliminating the background field $\hat{H}$ in the next section, i.e. in a diagrammatical procedure these terms contribute to reducible diagrams with

internal Higgs tree lines. Therefore, we do not have to calculate the counterterms completely, but only those contributions which yield $\mathcal{O}(M_{\rm H}^0)$ effects to the final Lagrangian. In particular, $\delta M_{\rm H}^2$ only has to be determined at $\mathcal{O}(M_{\rm H}^4)$, because $\hat{H}$ turns out to be $\mathcal{O}(M_{\rm H}^{-2})$ when it will be eliminated in the next section. For the same reason it is sufficient to consider $\delta M_{\rm W}^2$ only at $\mathcal{O}(M_{\rm H}^2)$.

As in Ref. [1], we call the sum of $\mathcal{L}_{\rm eff}$ (5.8) and $\mathcal{L}_{\rm \hat{H}}^{\rm ct}$ (6.9) the *renormalized effective Lagrangian* $\mathcal{L}_{\rm eff}^{\rm ren}$. Inserting (6.4), we find for the $\hat{H}$-dependent part of $\mathcal{L}_{\rm eff}^{\rm ren}$

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm ren}|_{\hat{H}} = \frac{1}{16\pi^2}\Bigg\{ & \frac{3g_2^2M_{\rm H}^4}{16M_{\rm W}^2}\Big(3I_{020} - 3B_0(M_{\rm H}^2, M_{\rm H}, M_{\rm H}) - \mathrm{Re}\Big\{B_0(M_{\rm H}^2,0,0)\Big\}\Big)\hat{H}^2 \\ & + \frac{g_2}{8M_{\rm W}}\Big(- I_{010} + 4I_{111}^W(1) + 3M_{\rm H}^2 I_{020} - 12M_{\rm H}^2 I_{121}\Big)\hat{H}\,\mathrm{tr}\Big\{\hat{V}_\mu\hat{V}^\mu\Big\}\Bigg\} \\ & + \mathcal{O}(\zeta^{-2}),\end{aligned} \tag{6.10}$$

while the $\hat{H}$-independent part is obviously the same as in (5.8).

7 Elimination of the background Higgs field

Having integrated out the quantum Higgs field H, which corresponds to Higgs lines in loops, the effective Lagrangian still contains the background Higgs field $\hat{H}$, which corresponds to Higgs tree lines in Feynman diagrams. The field $\hat{H}$ can now be eliminated in complete analogy to the procedure of Ref. [1] so that we discuss this point only briefly here. Since the $\hat{H}$-field corresponds to tree lines, the $\hat{H}$-propagators can be expanded in powers of $1/M_{\rm H}^2$ for $M_{\rm H}\to\infty$. Diagrammatically this means that the $\hat{H}$-propagator shrinks to a point rendering such (sub-)graphs irreducible which contain $\hat{H}$-lines only. The tree-level Lagrangian of the SM implies that this expansion corresponds to the replacement

$$\hat{H} \to -\frac{M_{\rm W}}{g_2 M_{\rm H}^2}\,\mathrm{tr}\Big\{\hat{V}_\mu\hat{V}^\mu\Big\} + \mathcal{O}(M_{\rm H}^{-4}). \tag{7.1}$$

The substitution (7.1) can be alternatively motivated by the fact that it corresponds to the use of the equation of motion (EOM) for the background Higgs field, which is fulfilled in lowest order by the tree-like part of Feynman diagrams. After applying (7.1), the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm ren}$ becomes:

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm ren} = \frac{1}{16\pi^2}\Bigg\{ & \Big(\frac{1}{4}I_{010} + M_{\rm W}^2 I_{011} - I_{111}^W(1)\Big)\,\mathrm{tr}\Big\{\hat{V}_\mu\hat{V}^\mu\Big\} \\ & + g_2^2\Big(-\frac{1}{2}I_{112} + 2I_{213}\Big)\,\mathrm{tr}\Big\{\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big\} \\ & + g_1^2\Big(- \frac{1}{4}I_{112} + I_{213}\Big)\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \\ & + \Big(\frac{1}{2}\frac{g_1^2}{g_2^2}I_{011} + \frac{1}{2M_{\rm W}^2}\Big[I_{111}^W(1) - I_{111}^Z(1)\Big]\Big)\mathcal{L}_0 \\ & + \frac{g_1}{g_2}\Big(- I_{112} + 4I_{213}\Big)\mathcal{L}_1\end{aligned}$$

$$
\begin{aligned}
&\quad + \frac{g_1}{g_2}\Big(- \frac{1}{2} I_{112} + 4 I_{213} \Big) \mathcal{L}_2 \\
&\quad + \Big(\frac{1}{2} I_{112} - 4 I_{213} \Big) \mathcal{L}_3 \\
&\quad + \Big(- 4 I_{213} + 2 I_{222} \Big) \mathcal{L}_4 \\
&\quad + \Big(\frac{1}{8 M_{\mathrm{H}}^2} I_{010} - \frac{1}{2 M_{\mathrm{H}}^2} I_{111}(1) + \frac{1}{4} I_{020} + I_{121} + 4 I_{213} + I_{222} \\
&\qquad\qquad - \frac{9}{16} B_0(M_{\mathrm{H}}^2, M_{\mathrm{H}}, M_{\mathrm{H}}) - \frac{3}{16} \operatorname{Re}\Big\{ B_0(M_{\mathrm{H}}^2, 0, 0) \Big\} \Big) \mathcal{L}_5 \\
&\quad + \Big(- I_{011} + 5 I_{112} - 12 I_{213} \Big) \mathcal{L}_{11} \Big\} \\
& + \mathcal{O}(M_{\mathrm{H}}^{-2}).
\end{aligned}
\tag{7.2}
$$

Finally, we insert the explicit forms (A.1) and (A.3) of the integrals in this expression and find:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}} = \frac{1}{16\pi^2} \Big\{ & \Big[- \frac{1}{8} M_{\mathrm{H}}^2 + \frac{3}{4} M_{\mathrm{W}}^2 \Big(\Delta_{M_{\mathrm{H}}} + \frac{5}{6} \Big) \Big] \operatorname{tr}\Big\{ \hat{V}_\mu \hat{V}^\mu \Big\} \\
& - \frac{1}{24} \Big(\Delta_{M_{\mathrm{H}}} + \frac{5}{6} \Big) g_2^2 \operatorname{tr}\Big\{ \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Big\} \\
& - \frac{1}{48} \Big(\Delta_{M_{\mathrm{H}}} + \frac{5}{6} \Big) g_1^2 \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \\
& + \frac{3}{8} \Big(\Delta_{M_{\mathrm{H}}} + \frac{5}{6} \Big) \frac{g_1^2}{g_2^2} \mathcal{L}_0 \\
& - \frac{1}{12} \Big(\Delta_{M_{\mathrm{H}}} + \frac{5}{6} \Big) \frac{g_1}{g_2} \mathcal{L}_1 \\
& + \frac{1}{24} \Big(\Delta_{M_{\mathrm{H}}} + \frac{17}{6} \Big) \frac{g_1}{g_2} \mathcal{L}_2 \\
& - \frac{1}{24} \Big(\Delta_{M_{\mathrm{H}}} + \frac{17}{6} \Big) \mathcal{L}_3 \\
& - \frac{1}{12} \Big(\Delta_{M_{\mathrm{H}}} + \frac{17}{6} \Big) \mathcal{L}_4 \\
& - \frac{1}{24} \Big(\Delta_{M_{\mathrm{H}}} + \frac{79}{3} - \frac{9\sqrt{3}\pi}{2} \Big) \mathcal{L}_5 \\
& - \frac{1}{4} \Big(\Delta_{M_{\mathrm{H}}} + \frac{1}{6} \Big) \mathcal{L}_{11} \Big\} \\
+ \mathcal{O}(M_{\mathrm{H}}^{-2}), &
\end{aligned}
\tag{7.3}
$$

with $\Delta_{M_{\mathrm{H}}}$ being given in (A.2).

The tree-level Lagrangian of the SM for $M_{\mathrm{H}} \to \infty$ is the Lagrangian of the corresponding $\mathrm{SU}(2)_{\mathrm{W}} \times \mathrm{U}(1)_{\mathrm{Y}}$ gauged non-linear σ-model (GNLSM) [18, 19], which is obtained from the SM Lagrangian simply be dropping the Higgs field in the non-linear realization of the scalar fields (2.8)

$$
\mathcal{L}^{\mathrm{tree}}|_{M_{\mathrm{H}} \to \infty} = \mathcal{L}^{\mathrm{tree}}|_{\hat{H}=0} + \mathcal{O}(M_{\mathrm{H}}^{-2}) = \mathcal{L}_{\mathrm{GNLSM}}^{\mathrm{tree}} + \mathcal{O}(M_{\mathrm{H}}^{-2}),
\tag{7.4}
$$

with

$$\mathcal{L}^{\text{tree}}_{\text{GNLSM}} = -\frac{1}{2}\,\text{tr}\left\{\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right\} - \frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{M_{\text{W}}^2}{g_2^2}\,\text{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\}. \tag{7.5}$$

The complete one-loop Lagrangian $\mathcal{L}^{1-\text{loop,ren}}\Big|_{M_{\text{H}}\to\infty}$ of the SM for $M_{\text{H}} \to \infty$ consists of three different parts: The effective Lagrangian $\mathcal{L}^{\text{ren}}_{\text{eff}}$, the part $\mathcal{L}^{1-\text{loop}}\Big|_{H=0}$ of the one-loop Lagrangian which does not contain the quantum Higgs field H, and the part $\mathcal{L}^{\text{ct}}|_{\hat{H}=0}$ of the countertem Lagrangian which does not contain the background field $\hat{H}$. As in Ref. [1], one can easily show that eliminating the background Higgs field $\hat{H}$ in $\mathcal{L}^{1-\text{loop}}\Big|_{H=0}$ by applying (7.1) simply results in dropping all terms which contain $\hat{H}$. Thus, we find that the one-loop Lagrangian of the SM for $M_{\text{H}} \to \infty$ is the sum of the one-loop Lagrangian of the GNLSM, the corresponding counterterm Lagrangian, and the effective Lagrangian

$$\begin{aligned}\mathcal{L}^{1-\text{loop,ren}}|_{M_{\text{H}}\to\infty} &= \mathcal{L}^{1-\text{loop}}|_{H=\hat{H}=0} + \mathcal{L}^{\text{ct}}|_{\hat{H}=0} + \mathcal{L}^{\text{ren}}_{\text{eff}} + \mathcal{O}(M_{\text{H}}^{-2}) \\ &= \mathcal{L}^{1-\text{loop}}_{\text{GNLSM}} + \mathcal{L}^{\text{ct}}_{\text{GNLSM}} + \mathcal{L}^{\text{ren}}_{\text{eff}} + \mathcal{O}(M_{\text{H}}^{-2}).\end{aligned} \tag{7.6}$$

The counterterm Lagrangian $\mathcal{L}^{\text{ct}}_{\text{GNLSM}}$ follows from the tree-level Lagrangian of the GNLSM (7.5) by applying the renormalization transformations (6.1) and (6.6). The renormalization constants occurring in $\mathcal{L}^{\text{ct}}_{\text{GNLSM}}$ are calculated from self-energies, as e.g. given in (6.3) for the mass and charge renormalization constants. Of course, the contribution of the effective Lagrangian $\mathcal{L}^{\text{ren}}_{\text{eff}}$ to the relevant self-energies have to be included in this procedure.

The first three terms in (7.3) have the same structure as terms in the tree-level Lagrangian of the GNLSM (7.5). They can be absorbed into the corresponding counterterms and have no effect on S-matrix elements. Furthermore, the $\mathcal{L}_{11}$-term in (7.3) does not affect S-matrix elements[5], because $\mathcal{L}_{11}$ (5.1) can be eliminated by applying the EOMs [20] for the $\text{SU}(2)_{\text{W}}$ background vector fields within the GNLSM [1],

$$\hat{D}^\mu_W\hat{W}_{\mu\nu} = -\frac{i}{g_2}M_{\text{W}}^2\hat{V}_\nu. \tag{7.7}$$

Using $\hat{D}^\mu_W\hat{D}^\nu_W\hat{W}_{\mu\nu} = 0$, this leads to

$$\hat{D}^\mu_W\hat{V}_\mu = 0, \tag{7.8}$$

which is valid at tree-level. Since $\mathcal{L}^{\text{ren}}_{\text{eff}}$ only contains background fields (corresponding to tree lines), this is sufficient to render the contribution of $\mathcal{L}_{11}$ to the S-matrix zero. Thus, the complete one-loop effects of a heavy Higgs boson on S-matrix elements, i.e. the complete difference between the SM for $M_{\text{H}} \to \infty$ and the GNLSM contributing to the S-matrix at one loop, are summarized in the effective Lagrangian

$$\mathcal{L}^{\text{ren}}_{\text{eff}}(\text{S-matrix}) = \frac{1}{16\pi^2}\Bigg\{ \quad \frac{3}{8}\left(\Delta_{M_{\text{H}}} + \frac{5}{6}\right)\frac{g_1^2}{g_2^2}M_{\text{W}}^2\left(\text{tr}\left\{\hat{T}\hat{V}_\mu\right\}\right)^2$$

[5] $\mathcal{L}_{11}$ yields contributions to S-matrix elements if massive fermions are included. This is discussed in the next section.

$$
\begin{aligned}
&- \frac{1}{24}\Big(\Delta_{M_\mathrm{H}} + \frac{5}{6}\Big) g_1 g_2 \hat{B}_{\mu\nu}\,\mathrm{tr}\left\{\hat{T}\hat{W}^{\mu\nu}\right\} \\
&+ \frac{1}{48}\Big(\Delta_{M_\mathrm{H}} + \frac{17}{6}\Big) i g_1 \hat{B}_{\mu\nu}\,\mathrm{tr}\left\{\hat{T}[\hat{V}^\mu,\hat{V}^\nu]\right\} \\
&- \frac{1}{24}\Big(\Delta_{M_\mathrm{H}} + \frac{17}{6}\Big) i g_2\,\mathrm{tr}\left\{\hat{W}_{\mu\nu}[\hat{V}^\mu,\hat{V}^\nu]\right\} \\
&- \frac{1}{12}\Big(\Delta_{M_\mathrm{H}} + \frac{17}{6}\Big)\left(\mathrm{tr}\left\{\hat{V}_\mu\hat{V}_\nu\right\}\right)^2 \\
&- \frac{1}{24}\Big(\Delta_{M_\mathrm{H}} + \frac{79}{3} - \frac{9\sqrt{3}\pi}{2}\Big)\left(\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\}\right)^2 \Bigg\} \\
&+ \mathcal{O}(M_\mathrm{H}^{-2}),
\end{aligned}
\tag{7.9}
$$

where the explicit form of the traces (5.1) is inserted.

Finally, we note that the result of our functional calculation (7.9) coincides with the result of the diagrammatical calculation in Ref. [6][6]. (Note that our coupling constants g_1 and g_2 correspond to the constants g' and g in Ref. [6] by the substitutions $g_1 \to g'$, $g_2 \to -g$.)

8 Fermionic contributions to the effective Lagrangian

8.1 The fermionic part of the standard model Lagrangian

In the previous sections we have only considered the bosonic sector of the electroweak SM. Now, we also include fermions in our calculation and determine the fermionic terms of the low-energy effective Lagrangian generated by integrating out the Higgs field.

The fermionic part of the SM Lagrangian is

$$
\mathcal{L}_F = i\left(\overline{\Psi}_f \not{D}_{f,\sigma}\omega_\sigma \Psi_f\right) - \frac{\sqrt{2}}{v}\left(\overline{\Psi}_f M_\mathrm{f}\Phi^\dagger\omega_-\Psi_f + \overline{\Psi}_f \Phi M_\mathrm{f}\omega_+\Psi_f\right), \tag{8.1}
$$

where the index f labels the different fermion doublets Ψ_f with the mass matrix[7] M_f, and $\omega_\pm$ denote the chirality projectors,

$$
\Psi_f = \begin{pmatrix} \psi_{f_1} \\ \psi_{f_2} \end{pmatrix}, \qquad M_\mathrm{f} = \begin{pmatrix} m_{\mathrm{f}_1} & 0 \\ 0 & m_{\mathrm{f}_2} \end{pmatrix}, \qquad \omega_\pm = \frac{1}{2}(1 \pm \gamma_5). \tag{8.2}
$$

In (8.1) and the following summation over all doublets Ψ_f is assumed. The covariant derivatives are

$$
D^\mu_{f,\sigma} = \partial^\mu - i g_2 W^\mu \delta_{\sigma-} + \frac{i}{2} g_1 Y_{\mathrm{f},\sigma} B^\mu \tag{8.3}
$$

[6]We find a coefficient for the $\mathcal{L}_{11}$-term in (7.3) which is different from the one in Ref. [6]. This is due to the fact that we use the non-linear parametrization of the Higgs sector (2.6) while in Ref. [6] the linear one (2.4) is used. Such a reparametrization of the scalar fields may change Green functions but not S-matrix elements [13, 15]. As pointed out, the $\mathcal{L}_{11}$-term has no impact on S-matrix elements (as far as one considers the pure bosonic sector).

[7]We neglect quark mixing throughout, i.e. the CKM matrix is set to the unit matrix. The generalization to finite quark mixing is straightforward.

with

$$Y_{\mathrm{f},\sigma} = 2Q_{\mathrm{f}} - \tau_3\delta_{\sigma-} \tag{8.4}$$

where Q_{f} is the electric charge matrix of Ψ_f, and $Y_{\mathrm{f},\sigma}$ the weak hypercharge matrix of $\omega_\sigma\Psi_f$. The scalar field Φ is again non-linearly realized according to (2.6).

The BFM is applied by splitting the fermion fields linearly according to

$$\Psi_f \to \hat{\Psi}_f + \Psi_f, \qquad \overline{\Psi}_f \to \hat{\overline{\Psi}}_f + \overline{\Psi}_f, \tag{8.5}$$

and the boson fields according to (2.9). Finally, the Stueckelberg transformation of the fermion fields [14, 15]

$$\begin{array}{llll} \omega_-\hat{\Psi}_f \to \hat{U}\omega_-\hat{\Psi}_f, & \omega_-\Psi_f \to \hat{U}\omega_-\Psi_f, & \hat{\overline{\Psi}}_f\omega_+ \to \hat{\overline{\Psi}}_f\omega_+\hat{U}^\dagger, & \overline{\Psi}_f\omega_+ \to \overline{\Psi}_f\omega_+\hat{U}^\dagger, \\ \omega_+\hat{\Psi}_f \to \omega_+\hat{\Psi}_f, & \omega_+\Psi_f \to \omega_+\Psi_f, & \hat{\overline{\Psi}}_f\omega_- \to \hat{\overline{\Psi}}_f\omega_-, & \overline{\Psi}_f\omega_- \to \overline{\Psi}_f\omega_- \end{array} \tag{8.6}$$

together with the one of the bosons (2.15) removes the background Goldstone fields from the Lagrangian.

8.2 Diagonalization

The one-loop part of Lagrangian (8.1) can be written in the symbolic form

$$\begin{aligned} \mathcal{L}_F^{1-\mathrm{loop}} = \quad & \overline{\Psi}_f\Delta_f\Psi_f - \operatorname{tr}\{\varphi\delta\Delta_\varphi\varphi\} + H\operatorname{tr}\{\delta X_{H\varphi}\varphi\} + H\overline{\Psi}_f X_{fH} + H\overline{X}_{fH}\Psi_f \\ & + \overline{\Psi}_f M_{\mathrm{f}}\varphi\omega_- X^L_{f\varphi} + \overline{X}^L_{f\varphi}\omega_+\varphi M_{\mathrm{f}}\Psi_f + \overline{\Psi}_f\varphi M_{\mathrm{f}}\omega_+ X^R_{f\varphi} + \overline{X}^R_{f\varphi}\omega_- M_{\mathrm{f}}\varphi\Psi_f \\ & + \overline{\Psi}_f \not{W} X_{fW} + \overline{X}_{fW}\not{W}\Psi_f + \overline{\Psi}_f\not{B} X_{fB} + \overline{X}_{fB}\not{B}\Psi_f, \end{aligned} \tag{8.7}$$

with the operators

$$\begin{aligned} \Delta_f &= i\hat{\not{D}}_{f,\sigma}\omega_\sigma - M_{\mathrm{f}}\left(1 + \frac{\hat{H}}{v}\right), \\ \delta\Delta_\varphi &= -\frac{g_2^2}{4M_{\mathrm{W}}^2}\hat{\overline{\Psi}}_f M_{\mathrm{f}}\hat{\Psi}_f\left(1 + \frac{\hat{H}}{v}\right), \\ \delta X^{ab}_{H\varphi} &= -i\frac{g_2^2}{2M_{\mathrm{W}}^2}\left[\hat{\overline{\Psi}}{}^b_f\omega_+\left(M_{\mathrm{f}}\hat{\Psi}_f\right)^a - \left(\hat{\overline{\Psi}}_f M_{\mathrm{f}}\right)^b\omega_-\hat{\Psi}^a_f\right], \\ X_{fH} &= -\frac{g_2}{2M_{\mathrm{W}}}M_{\mathrm{f}}\hat{\Psi}_f, \\ X^L_{f\varphi} &= i\frac{g_2}{M_{\mathrm{W}}}\hat{\Psi}_f\left(1 + \frac{\hat{H}}{v}\right), \qquad X^R_{f\varphi} = -i\frac{g_2}{M_{\mathrm{W}}}\hat{\Psi}_f\left(1 + \frac{\hat{H}}{v}\right), \\ X_{fW} &= g_2\omega_-\hat{\Psi}_f, \qquad X_{fB} = -\frac{g_1}{2}Y_{\mathrm{f},\sigma}\omega_\sigma\hat{\Psi}_f. \end{aligned} \tag{8.8}$$

The indices a and b in the third line denote the $\mathrm{SU}(2)_{\mathrm{W}}$ indices of the 2×2-matrix $\delta X_{H\varphi}$.

As in Sect. 3, the mixings between the quantum Higgs field H and the other quantum fields can be removed by appropriate shifts of the quantum fields. It turns out to be useful

first to remove the $H\Psi_f$-mixing in (8.7) before diagonalizing the bosonic sector of the SM Lagrangian (3.3). This can be achieved by the shifts

$$\Psi_f \to \Psi_f - \Delta_f^{-1} X_{fH} H, \qquad \overline{\Psi}_f \to \overline{\Psi}_f - H\overline{X}_{fH}\overline{\Delta_f^{-1}} \tag{8.9}$$

with

$$\overline{\Delta_f^{-1}} = \gamma_0 \left(\Delta_f^{-1}\right)^\dagger \gamma_0, \tag{8.10}$$

which modify the term bilinear in H and the $H\varphi$-terms in (3.3) and (8.7) according to

$$\Delta_H \to \Delta_H + \delta\Delta_H, \qquad X_{H\varphi} + \delta X_{H\varphi} \to X_{H\varphi} + \delta X_{H\varphi} + \delta X'_{H\varphi} \tag{8.11}$$

with

$$\begin{aligned} \delta\Delta_H &= 2\overline{X}_{fH}\Delta_f^{-1}X_{fH} \\ H\,\mathrm{tr}\left\{\delta X'_{H\varphi}\varphi\right\} &= -H\overline{X}_{fH}\left(\Delta_f^{-1}M_\mathrm{f}\varphi\omega_- X^L_{f\varphi}\right) - H\overline{X}_{fH}\left(\Delta_f^{-1}\varphi M_\mathrm{f}\omega_+ X^R_{f\varphi}\right) \\ &\quad - \overline{X}^L_{f\varphi}\omega_+\varphi M_\mathrm{f}\left(\Delta_f^{-1}X_{fH}H\right) - \overline{X}^R_{f\varphi}\omega_- M_\mathrm{f}\varphi\left(\Delta_f^{-1}X_{fH}H\right). \end{aligned} \tag{8.12}$$

In (8.12), we define $\delta X'_{H\varphi}$ implicitly via $H\,\mathrm{tr}\left\{\delta X'_{H\varphi}\varphi\right\}$ since its explicit expression outside the trace is not needed in the following. In addition to (8.11), there is a modification of the HW- and HB-terms, which however can be neglected at $\mathcal{O}(M_\mathrm{H}^0)$. We also had to remove the $f\varphi$-, fW- and fB-terms by appropriate shifts before doing the shifts (3.5) in the bosonic sector (such that those do not effect the fermionic sector), and finally reverse these shifts in order to restore these terms. However, it turns out by simple power counting that the contributions of these shifts to the Δs and Xs in the bosonic sector (3.11) only yield $\mathcal{O}(M_\mathrm{H}^{-2})$-effects.

This means that all fermionic $\mathcal{O}(M_\mathrm{H}^0)$-contributions to $\mathcal{L}_\mathrm{eff}$ can be found by adding $\delta\Delta_H$, $\delta\Delta_\varphi$, $\delta X_{H\varphi}$ and $\delta X'_{H\varphi}$ given by (8.8) and (8.12) to the bosonic parameters (3.11), and proceeding as in the calculation of the bosonic part of $\mathcal{L}_\mathrm{eff}$. Thus, $\tilde{\tilde{\Delta}}_H$ (3.8) modifies to

$$\begin{aligned} \tilde{\tilde{\Delta}}_H &\to \tilde{\tilde{\Delta}}_H + \delta\tilde{\tilde{\Delta}}_H \\ &= \tilde{\tilde{\Delta}}_H + \delta\Delta_H - \frac{1}{2}\,\mathrm{tr}\left\{X_{H\varphi}\delta\left(\hat{\Delta}_\varphi^{-1}\right)X^\dagger_{H\varphi}\right\} - \frac{1}{2}\left(\mathrm{tr}\left\{\delta X_{H\varphi}\hat{\Delta}_\varphi^{-1}X^\dagger_{H\varphi}\right\} + \mathrm{h.c.}\right) \\ &\quad - \frac{1}{2}\,\mathrm{tr}\left\{\delta X_{H\varphi}\hat{\Delta}_\varphi^{-1}\delta X^\dagger_{H\varphi}\right\} - \frac{1}{2}\left(\mathrm{tr}\left\{\delta X'_{H\varphi}\hat{\Delta}_\varphi^{-1}X^\dagger_{H\varphi}\right\} + \mathrm{h.c.}\right) + \mathcal{O}(\zeta^{-3}) \end{aligned} \tag{8.13}$$

with

$$\delta\left(\hat{\Delta}_\varphi^{-1}\right) = \left(\widehat{\Delta_\varphi + \delta\Delta_\varphi}\right)^{-1} - \hat{\Delta}_\varphi^{-1}. \tag{8.14}$$

In (8.13) terms yielding only $\mathcal{O}(M_\mathrm{H}^{-2})$-contributions are again neglected.

8.3 $1/M_\mathrm{H}$-Expansion

The fermionic part of $\mathcal{L}_\mathrm{eff}$ can be derived by expanding the contribution of $\delta\tilde{\tilde{\Delta}}_H$ in (8.13) to (4.1) in analogy to the procedure described in Sect. 4. This yields

$$\delta\mathcal{L}_\mathrm{eff} = \frac{1}{16\pi^2}\Bigg\{ \quad \frac{g_2^2}{4M_\mathrm{W}^2}I_{011}\hat{\overline{\Psi}}_f M_\mathrm{f}^3\hat{\Psi}_f + \frac{ig_2^2}{4M_\mathrm{W}^2}\left(I_{011} - 2I_{112}\right)\hat{\overline{\Psi}}_f M_\mathrm{f}\hat{\not{D}}_{f,\sigma}M_\mathrm{f}\omega_{-\sigma}\hat{\Psi}_f$$

$$
\begin{aligned}
&- \frac{g_2^3}{2M_{\rm W}^2} I_{112} \hat{\bar{\Psi}}_f \left[2M_{\rm f}\hat{\not{C}}M_{\rm f}\omega_+ - \left(M_{\rm f}^2\hat{\not{C}} + \hat{\not{C}}M_{\rm f}^2 \right)\omega_- \right] \hat{\Psi}_f \\
&- \frac{g_2^4}{4M_{\rm W}^2} I_{112} \hat{\bar{\Psi}}_f M_{\rm f} \hat{\Psi}_f \,{\rm tr}\left\{ \hat{C}_\mu \hat{C}^\mu \right\} \\
&- \frac{ig_2^3}{2M_{\rm W}^2} \left(I_{011} - 2I_{112} \right) \hat{\bar{\Psi}}_f \left[\left(\hat{D}_W^\mu \hat{C}_\mu \right) M_{\rm f}\omega_+ - M_{\rm f} \left(\hat{D}_W^\mu \hat{C}_\mu \right)\omega_- \right] \hat{\Psi}_f \\
&- \frac{g_2^4}{32M_{\rm W}^4} I_{011} \left[\hat{\bar{\Psi}}_f \left(\tau_i M_{\rm f}\omega_+ - M_{\rm f}\tau_i\omega_- \right) \hat{\Psi}_f \right] \\
&\qquad\qquad\qquad \times \left[\hat{\bar{\Psi}}_{f'} \left(\tau_i M_{\rm f'}\omega_+ - M_{\rm f'}\tau_i\omega_- \right) \hat{\Psi}_{f'} \right] \Bigg\} \\
&+ \mathcal{O}(\zeta^{-2}).
\end{aligned}
\tag{8.15}
$$

Strictly speaking, in (8.15) vacuum integrals of the form

$$
\frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^Dp \frac{p_{\mu_1} \dots p_{\mu_{2k}}}{(p^2 - M_{\rm H}^2)^l (p^2 - M_1^2)^{m_1} (p^2 - M_2^2)^{m_2}} \quad \text{with} \quad M_{1,2}^2 = \xi M_{\rm W,Z}^2, m_{{\rm f}_i}^2 \tag{8.16}
$$

occur, because in addition to the bosonic propagators there are also fermionic ones. Since in (8.15) only logarithmically divergent integrals are relevant, which are independent of M_1^2 and M_2^2 (and thus depend only on $m = m_1 + m_2$) at $\mathcal{O}(M_{\rm H}^0)$, these are still given by the explicit expressions (A.1) for the integrals I_{klm} (4.4). In particular, the fact that the fermion masses within a doublet can be different does not effect these integrals at $\mathcal{O}(M_{\rm H}^0)$.

The origin of the various terms in $\mathcal{L}_{\rm eff}$ (8.15) is the following: the first two terms are the contribution of $\delta\Delta_H$ in (8.13), the third term is the contribution of $\delta X'_{H\varphi}\hat{\Delta}_\varphi^{-1}X_{H\varphi}^\dagger + \text{h.c.}$, the fourth stems from $X_{H\varphi}\delta\left(\hat{\Delta}_\varphi^{-1}\right)X_{H\varphi}^\dagger$, the fifth from $\delta X_{H\varphi}\hat{\Delta}_\varphi^{-1}X_{H\varphi}^\dagger + \text{h.c.}$, and the last from $\delta X_{H\varphi}\hat{\Delta}_\varphi^{-1}\delta X_{H\varphi}^\dagger$. Note that the explicit occurrence of the Pauli matrices τ_i in the last term in (8.15) is a consequence of the operator P (3.10) in $\hat{\Delta}_\varphi^{-1}(x, \partial_x + ip)$ (4.8).

8.4 The Stueckelberg formalism

We invert the Stueckelberg transformation (2.15), (8.6) in order to rewrite $\delta\mathcal{L}_{\rm eff}$ in a gauge-invariant form. The inverse Stueckelberg transformation is given by (5.6) and

$$
\omega_-\hat{\Psi}_f \to \hat{U}^\dagger\omega_-\hat{\Psi}_f, \qquad \hat{\bar{\Psi}}_f\omega_+ \to \hat{\bar{\Psi}}_f\omega_+\hat{U}, \qquad \omega_+\hat{\Psi}_f \to \omega_+\hat{\Psi}_f, \qquad \hat{\bar{\Psi}}_f\omega_- \to \hat{\bar{\Psi}}_f\omega_-. \tag{8.17}
$$

This yields

$$
\begin{aligned}
\delta\mathcal{L}_{\rm eff} = \frac{1}{16\pi^2}\Bigg\{ \quad & \frac{g_2^2}{4M_{\rm W}^2} I_{011} \hat{\bar{\Psi}}_f \left(\hat{U}M_{\rm f}^3\omega_+ + M_{\rm f}^3\hat{U}^\dagger\omega_- \right) \hat{\Psi}_f \\
&+ \frac{ig_2^2}{4M_{\rm W}^2} \left(I_{011} - 2I_{112} \right) \hat{\bar{\Psi}}_f \left(M_{\rm f}\hat{U}^\dagger \hat{\not{D}}_{f,-}\hat{U}M_{\rm f}\omega_+ + \hat{U}M_{\rm f}\hat{\not{D}}_{f,+}M_{\rm f}\hat{U}^\dagger\omega_- \right) \hat{\Psi}_f \\
&- \frac{ig_2^2}{2M_{\rm W}^2} I_{112} \hat{\bar{\Psi}}_f \left[2M_{\rm f}\hat{U}^\dagger\hat{\not{V}}\hat{U}M_{\rm f}\omega_+ - \left(\hat{U}M_{\rm f}^2\hat{U}^\dagger\hat{\not{V}} + \hat{\not{V}}\hat{U}M_{\rm f}^2\hat{U}^\dagger \right)\omega_- \right] \hat{\Psi}_f
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \frac{g_2^2}{4M_\mathrm{W}^2} I_{112} \bar{\hat{\Psi}}_f \left(\hat{U} M_\mathrm{f} \omega_+ + M_\mathrm{f} \hat{U}^\dagger \omega_- \right) \hat{\Psi}_f \,\mathrm{tr}\left\{ \hat{V}_\mu \hat{V}^\mu \right\} \\
&\quad + \frac{g_2^2}{2M_\mathrm{W}^2} \left(I_{011} - 2I_{112} \right) \bar{\hat{\Psi}}_f \left[\left(\hat{D}_W^\mu \hat{V}_\mu \right) \hat{U} M_\mathrm{f} \omega_+ - M_\mathrm{f} \hat{U}^\dagger \left(\hat{D}_W^\mu \hat{V}_\mu \right) \omega_- \right] \hat{\Psi}_f \\
&\quad - \frac{g_2^4}{32M_\mathrm{W}^4} I_{011} \left[\bar{\hat{\Psi}}_f \left(\hat{U} \tau_i M_\mathrm{f} \omega_+ - M_\mathrm{f} \tau_i \hat{U}^\dagger \omega_- \right) \hat{\Psi}_f \right] \\
&\qquad\qquad \times \left[\bar{\hat{\Psi}}_{f'} \left(\hat{U} \tau_i M_{\mathrm{f}'} \omega_+ - M_{\mathrm{f}'} \tau_i \hat{U}^\dagger \omega_- \right) \hat{\Psi}_{f'} \right] \Bigg\} \\
&+ \mathcal{O}(\zeta^{-2}).
\end{aligned}
\tag{8.18}
$$

This Lagrangian is invariant under the background gauge transformations (2.12) and

$$
\begin{aligned}
\omega_- \hat{\Psi}_f &\to S S_{\mathrm{Y}_{\mathrm{f},-}} \omega_- \hat{\Psi}_f, \qquad & \omega_+ \hat{\Psi}_f &\to S_{\mathrm{Y}_{\mathrm{f},+}} \omega_+ \hat{\Psi}_f, \\
\bar{\hat{\Psi}}_f \omega_+ &\to \bar{\hat{\Psi}}_f \omega_+ S^\dagger_{\mathrm{Y}_{\mathrm{f},-}} S^\dagger, \qquad & \bar{\hat{\Psi}}_f \omega_- &\to \bar{\hat{\Psi}}_f \omega_- S^\dagger_{\mathrm{Y}_{\mathrm{f},+}},
\end{aligned}
\tag{8.19}
$$

where S is given by (2.13) and $S_{\mathrm{Y}_{\mathrm{f},\sigma}}$ by

$$
S_{\mathrm{Y}_{\mathrm{f},\sigma}} = \exp\left(-\frac{i}{2} g_1 Y_{\mathrm{f},\sigma} \theta_\mathrm{Y} \right) \tag{8.20}
$$

with the weak hypercharges $Y_{\mathrm{f},\sigma}$ (8.4).

The second term in (8.18) can be simplified by applying the product rule for the covariant derivatives. This yields a term with derivatives acting only on the fermion fields and a term which has the same structure as the third term in (8.18). The dimension-4 part (i.e. the second and third term) of (8.18) becomes

$$
\begin{aligned}
\delta\mathcal{L}_\mathrm{eff}\big|_{\mathrm{dim}=4} = \frac{1}{16\pi^2} \Bigg\{ &\quad \frac{ig_2^2}{8M_\mathrm{W}^2} \left(I_{011} - 2I_{112} \right) \left[\bar{\hat{\Psi}}_f \left(M_\mathrm{f}^2 \hat{\not{D}}_{f,+} \omega_+ + \hat{U} M_\mathrm{f}^2 \hat{U}^\dagger \hat{\not{D}}_{f,-} \omega_- \right) \hat{\Psi}_f + \mathrm{h.c.} \right] \\
&+ \frac{ig_2^2}{8M_\mathrm{W}^2} (I_{011} - 6I_{112}) \bar{\hat{\Psi}}_f \Big[2M_\mathrm{f} \hat{U}^\dagger \hat{\not{V}} \hat{U} M_\mathrm{f} \omega_+ \\
&\qquad - \left(\hat{U} M_\mathrm{f}^2 \hat{U}^\dagger \hat{\not{V}} + \hat{\not{V}} \hat{U} M_\mathrm{f}^2 \hat{U}^\dagger \right) \omega_- \Big] \hat{\Psi}_f \Bigg\}.
\end{aligned}
\tag{8.21}
$$

8.5 Renormalization

In analogy to Sect. 6, we have to add the fermionic part of the Higgs dependent counterterms to $\delta\mathcal{L}_\mathrm{eff}$. The parameter- and field-renormalization transformations of the fermions are

$$
\begin{aligned}
m_{\mathrm{f}_i} &\to m_{\mathrm{f}_i,0} = m_{\mathrm{f}_i} + \delta m_{\mathrm{f}_i}, \\
\omega_\sigma \hat{\psi}_{f_i} &\to \omega_\sigma \hat{\psi}_{f_i,0} = (Z^\sigma_{f_i})^{1/2} \omega_\sigma \hat{\psi}_{f_i} = (1 + \frac{1}{2} \delta Z^\sigma_{f_i}) \omega_\sigma \hat{\psi}_{f_i}.
\end{aligned}
\tag{8.22}
$$

From (8.18) one immediately reads

$$
\frac{\delta m_{\mathrm{f}_i}}{m_{\mathrm{f}_i}} = \mathcal{O}(M_\mathrm{H}^0), \qquad \delta Z^\sigma_{f_i} = \mathcal{O}(M_\mathrm{H}^0). \tag{8.23}
$$

In this context, one should notice that the renormalized effective action only remains gauge-invariant if the left-handed fermion-doublet field $\omega_-\Psi_f$ is renormalized by *one* renormalization constant, i.e. $\delta Z_f^L = \delta Z_{f_1}^L = \delta Z_{f_2}^L$ (in δZ_f the superscripts R/L are used instead of $\sigma = +/-$). Similarly to the case of the gauge-boson fields considered in Sect. 6, the explicit form of the field-renormalization constants $\delta Z_{f_i}^\sigma$ is irrelevant for the construction of the effective Lagrangian as long as (8.23) holds. In particular, (8.23) is fulfilled in the complete on-shell scheme [17], where all fermion propagators acquire residues equal to one. According to simple power counting, we only have to consider the contribution of δM_W^2 to $\delta\mathcal{L}_{\hat{\mathrm{H}}}^{\mathrm{ct}}$:

$$\delta\mathcal{L}_{\hat{\mathrm{H}}}^{\mathrm{ct}} = \frac{g_2}{4M_\mathrm{W}^3}\delta M_\mathrm{W}^2 \hat{H}\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_+ + M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f + \mathcal{O}(\zeta^{-2}), \tag{8.24}$$

with δM_W given in (6.4). The fermionic part $\delta\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}}$ of the renormalized effective Lagrangian is the sum of $\delta\mathcal{L}_{\mathrm{eff}}$ (8.18) and $\delta\mathcal{L}_{\hat{\mathrm{H}}}^{\mathrm{ct}}$ (8.24).

8.6 Elimination of the background Higgs field

As in Sect. 7, we can eliminate the background Higgs field $\hat{H}$ by a propagator expansion, or equivalently by an application of the EOM for $\hat{H}$ in lowest order. The fermionic part of the SM Lagrangian (8.1) implies that (7.1) generalizes to

$$\hat{H} \to -\frac{M_\mathrm{W}}{g_2 M_\mathrm{H}^2}\,\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\} - \frac{g_2}{2M_\mathrm{W}M_\mathrm{H}^2}\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_+ + M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f + \mathcal{O}(M_\mathrm{H}^{-4}). \tag{8.25}$$

Applying this to the complete effective Lagrangian (i.e. to the bosonic and to the fermionic part), we finally find

$$\begin{aligned}
\delta\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}} = \frac{1}{16\pi^2}\Bigg\{ \quad & \frac{g_2^2}{4M_\mathrm{W}^2}I_{011}\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}^3\omega_+ + M_\mathrm{f}^3\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f \\
& + \frac{ig_2^2}{8M_\mathrm{W}^2}\left(I_{011} - 2I_{112}\right)\left[\hat{\overline{\Psi}}_f\left(M_\mathrm{f}^2\hat{\not{D}}_{f,+}\omega_+ + \hat{U}M_\mathrm{f}^2\hat{U}^\dagger\hat{\not{D}}_{f,-}\omega_-\right)\hat{\Psi}_f + \mathrm{h.c.}\right] \\
& + \frac{ig_2^2}{8M_\mathrm{W}^2}(I_{011} - 6I_{112})\hat{\overline{\Psi}}_f\Big[2M_\mathrm{f}\hat{U}^\dagger\hat{\not{V}}\hat{U}M_\mathrm{f}\omega_+ - \\
& \qquad\qquad \left(\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\hat{\not{V}} + \hat{\not{V}}\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\right)\omega_-\Big]\hat{\Psi}_f \\
& + \frac{g_2^2}{M_\mathrm{W}^2}\left(\frac{3}{8}I_{020} + \frac{1}{4}I_{112} + \frac{3}{4}I_{121} - \frac{9}{16}B_0(M_\mathrm{H}^2, M_\mathrm{H}, M_\mathrm{H})\right. \\
& \qquad \left. - \frac{3}{16}\mathrm{Re}\left\{B_0(M_\mathrm{H}^2, 0, 0)\right\}\right)\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_+ + M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\,\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\} \\
& + \frac{g_2^2}{2M_\mathrm{W}^2}\left(I_{011} - 2I_{112}\right)\hat{\overline{\Psi}}_f\left[\left(\hat{D}_W^\mu\hat{V}_\mu\right)\hat{U}M_\mathrm{f}\omega_+ - M_\mathrm{f}\hat{U}^\dagger\left(\hat{D}_W^\mu\hat{V}_\mu\right)\omega_-\right]\hat{\Psi}_f \\
& - \frac{g_2^4}{32M_\mathrm{W}^4}I_{011}\left[\hat{\overline{\Psi}}_f\left(\hat{U}\tau_i M_\mathrm{f}\omega_+ - M_\mathrm{f}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right] \\
& \qquad\qquad \times\left[\hat{\overline{\Psi}}_{f'}\left(\hat{U}\tau_i M_{\mathrm{f}'}\omega_+ - M_{\mathrm{f}'}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{g_2^4}{8M_\mathrm{W}^4}\Big(-\frac{1}{4M_\mathrm{H}^2}I_{010}+\frac{1}{M_\mathrm{H}^2}I_{111}^W(1)+\frac{9}{8}I_{020}-\frac{9}{8}B_0(M_\mathrm{H}^2,M_\mathrm{H},M_\mathrm{H})\\
&\qquad-\frac{3}{8}\mathrm{Re}\Big\{B_0(M_\mathrm{H}^2,0,0)\Big)\Big\}\left[\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_++M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right]\\
&\qquad\times\left[\hat{\overline{\Psi}}_{f'}\left(\hat{U}M_{\mathrm{f}'}\omega_++M_{\mathrm{f}'}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right]\Bigg\}\\
&+\mathcal{O}(M_\mathrm{H}^{-2}).
\end{aligned}
\tag{8.26}
$$

With the explicit expressions for the integrals (A.1) this becomes

$$
\begin{aligned}
\delta\mathcal{L}_\mathrm{eff}^\mathrm{ren}=\frac{1}{16\pi^2}\Bigg\{\quad&\frac{1}{4}\left(\Delta_{M_\mathrm{H}}+1\right)\frac{g_2^2}{M_\mathrm{W}^2}\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}^3\omega_++M_\mathrm{f}^3\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\\
&+\frac{1}{16}\left(\Delta_{M_\mathrm{H}}+\frac{1}{2}\right)\frac{ig_2^2}{M_\mathrm{W}^2}\left[\hat{\overline{\Psi}}_f\left(M_\mathrm{f}^2\hat{\not{D}}_{f,+}\omega_++\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\hat{\not{D}}_{f,-}\omega_-\right)\hat{\Psi}_f+\text{h.c.}\right]\\
&-\frac{1}{16}\left(\Delta_{M_\mathrm{H}}+\frac{5}{2}\right)\frac{ig_2^2}{M_\mathrm{W}^2}\hat{\overline{\Psi}}_f\Big[2M_\mathrm{f}\hat{U}^\dagger\hat{\not{V}}\hat{U}M_\mathrm{f}\omega_+-\\
&\qquad\left(\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\hat{\not{V}}+\hat{\not{V}}\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\right)\omega_-\Big]\hat{\Psi}_f\\
&-\frac{1}{8}\left(\Delta_{M_\mathrm{H}}+\frac{21}{2}-\frac{3\sqrt{3}\pi}{2}\right)\frac{g_2^2}{M_\mathrm{W}^2}\hat{\overline{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_++M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\,\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\}\\
&+\frac{1}{4}\left(\Delta_{M_\mathrm{H}}+\frac{1}{2}\right)\frac{g_2^2}{M_\mathrm{W}^2}\hat{\overline{\Psi}}_f\left[\left(\hat{D}_W^\mu\hat{V}_\mu\right)\hat{U}M_\mathrm{f}\omega_+-M_\mathrm{f}\hat{U}^\dagger\left(\hat{D}_W^\mu\hat{V}_\mu\right)\omega_-\right]\hat{\Psi}_f\\
&-\frac{1}{32}\left(\Delta_{M_\mathrm{H}}+1\right)\frac{g_2^4}{M_\mathrm{W}^4}\left[\hat{\overline{\Psi}}_f\left(\hat{U}\tau_iM_\mathrm{f}\omega_+-M_\mathrm{f}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right]\\
&\qquad\times\left[\hat{\overline{\Psi}}_{f'}\left(\hat{U}\tau_iM_{\mathrm{f}'}\omega_+-M_{\mathrm{f}'}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right]\\
&-\frac{3}{64}\left(\Delta_{M_\mathrm{H}}+\frac{23}{3}-\sqrt{3}\pi\right)\frac{g_2^4}{M_\mathrm{W}^4}\left[\hat{\Psi}_f\left(\hat{U}M_\mathrm{f}\omega_++M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\overline{\Psi}}_f\right]\\
&\qquad\times\left[\hat{\Psi}_{f'}\left(\hat{U}M_{\mathrm{f}'}\omega_++M_{\mathrm{f}'}\hat{U}^\dagger\omega_-\right)\hat{\overline{\Psi}}_{f'}\right]\Bigg\}\\
+\mathcal{O}(M_\mathrm{H}^{-2}).&
\end{aligned}
\tag{8.27}
$$

8.7 Equations of motion and S-matrix

The tree-level and one-loop Lagrangian of the SM for $M_\mathrm{H}\to\infty$ are given by (7.4) and (7.6), respectively. The fermionic part of the GNLSM Lagrangian is derived from the SM Lagrangian (8.1) by dropping the Higgs field in the non-linear parametrization (2.6):

$$
\mathcal{L}_{\mathrm{GNLSM},F}=i\left(\overline{\Psi}_f\not{D}_{f,\sigma}\omega_\sigma\Psi_f\right)-\left(\overline{\Psi}_fM_\mathrm{f}U^\dagger\omega_-\Psi_f+\overline{\Psi}_fUM_\mathrm{f}\omega_+\Psi_f\right).
\tag{8.28}
$$

The first term in (8.27) has the same structure as the Yukawa term in the GNLSM Lagrangian (8.28). Since the masses of the fermion doublet are renormalized independently, this term can be absorbed into the corresponding counterterm, and thus it does not contribute to the S-matrix.

Next, we consider the second line in (8.27) which is related to the kinetic term in (8.28). The ω_+-part can be completely absorbed into the counterterm to the kinetic terms for the right-handed fermion fields since these are renormalized independently. For the ω_--part it is useful to decompose M_f (8.2) as [21]

$$M_\mathrm{f} = \frac{1}{2}(m_{\mathrm{f}_1} + m_{\mathrm{f}_2})\mathbf{1} + \frac{1}{2}(m_{\mathrm{f}_1} - m_{\mathrm{f}_2})\tau_3 \tag{8.29}$$

and M_f^2 accordingly. The contribution proportional to the unit matrix inserted into the ω_--term yields a term, which can be absorbed into the kinetic term of the left-handed fermion doublet. Thus, the only part of the second line in (8.27) which contributes to the S-Matrix is

$$\delta\mathcal{L}_\mathrm{eff}^\mathrm{ren}(\text{S-Matrix})|_{\hat{\not{D}}\hat{\Psi}_f} = \frac{1}{16\pi^2}\frac{1}{32}\left(\Delta_{M_\mathrm{H}} + \frac{1}{2}\right)\frac{ig_2^2}{M_\mathrm{W}^2}\left(m_{\mathrm{f}_1}^2 - m_{\mathrm{f}_2}^2\right)\left[\hat{\overline{\Psi}}_f \hat{T}\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f + \text{h.c.}\right], \tag{8.30}$$

with $\hat{T}$ defined in (5.2).

Finally, we may use the classical EOMs for the background fields in order to remove the $\hat{D}_W^\mu \hat{V}_\mu$-terms in $\mathcal{L}_\mathrm{eff}^\mathrm{ren}$. Such an application of the EOM within the effective interaction term corresponds to a shift of the background fields which does not effect S-matrix elements [20]. Relation (7.7) was derived for the pure bosonic sector of the SM. Taking into account massive fermions, the EOM for the $\mathrm{SU(2)_W}$ gauge fields within the GNLSM become

$$\hat{D}_W^\mu \hat{W}_{\mu\nu} = -\frac{i}{g_2}M_\mathrm{W}^2\hat{V}_\nu + PA_{1,\nu} \quad \text{with} \quad A_{1,\nu}^{ab} = -\frac{g_2}{2}\hat{\overline{\Psi}}_f^b\gamma_\nu\omega_-\hat{\Psi}_f^a, \tag{8.31}$$

and (7.8) generalizes to

$$\hat{D}_W^\mu \hat{V}_\mu = PA_2 \quad \text{with} \quad A_2^{ab} = \frac{ig_2^2}{2M_\mathrm{W}^2}\left[\left(\overline{\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f}\right)^b\hat{\Psi}_f^a + \hat{\overline{\Psi}}_f^b\left(\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f\right)^a\right], \tag{8.32}$$

where P is the operator defined in (3.10). In (8.31) and (8.32) and the following, the indices a and b denote the $\mathrm{SU(2)_W}$ indices of the 2×2-matrices A_i. Then, we can apply the EOMs for the fermion fields within the GNLSM

$$\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f = -i\hat{U}M_\mathrm{f}\omega_+\hat{\Psi}_f, \qquad \overline{\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f} = i\hat{\overline{\Psi}}_f\omega_-M_\mathrm{f}\hat{U}^\dagger, \tag{8.33}$$

and find

$$\hat{D}_W^\mu \hat{V}_\mu = PA_3 \quad \text{with} \quad A_3^{ab} = \frac{g_2^2}{2M_\mathrm{W}^2}\left[\hat{\overline{\Psi}}_f^b\omega_+\left(\hat{U}M_\mathrm{f}\hat{\Psi}_f\right)^a - \left(\hat{\overline{\Psi}}_f M_\mathrm{f}\hat{U}^\dagger\right)^b\omega_-\hat{\Psi}_f^a\right]. \tag{8.34}$$

Applying this to the $\hat{D}_W^\mu \hat{V}_\mu$-term in (8.27) one finds

$$\begin{aligned}
&\hat{\overline{\Psi}}_f\left[\left(\hat{D}_W^\mu \hat{V}_\mu\right)\hat{U}M_\mathrm{f}\omega_+ - M_\mathrm{f}\hat{U}^\dagger\left(\hat{D}_W^\mu \hat{V}_\mu\right)\omega_-\right]\hat{\Psi}_f \\
&= \frac{g_2^2}{4M_\mathrm{W}^2}\left[\hat{\overline{\Psi}}_f\left(\hat{U}\tau_i M_\mathrm{f}\omega_+ - M_\mathrm{f}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right]\left[\hat{\overline{\Psi}}_{f'}\left(\hat{U}\tau_i M_{\mathrm{f}'}\omega_+ - M_{\mathrm{f}'}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right],
\end{aligned} \tag{8.35}$$

and inserting this into $\mathcal{L}_{11}$ of (5.1), one obtains

$$\mathcal{L}_{11} = \frac{g_2^4}{8M_\mathrm{W}^4}\left[\bar{\hat{\Psi}}_f\left(\hat{U}\tau_i M_\mathrm{f}\omega_+ - M_\mathrm{f}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right]\left[\bar{\hat{\Psi}}_{f'}\left(\hat{U}\tau_i M_{\mathrm{f}'}\omega_+ - M_{\mathrm{f}'}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right]. \quad (8.36)$$

To derive (8.35) and (8.36), we have used the definition (3.10) and the identity

$$\mathrm{tr}\left\{(PAU)(PBU)\right\} = \mathrm{tr}\left\{(PUA)(PUB)\right\} \quad (8.37)$$

where A and B are arbitrary 2×2-matrices and U is an SU(2) matrix. Equation (8.37) is proven in App. B. Thus, if one considers massive fermions, the contribution of $\mathcal{L}_{11}$ to S-matrix elements does not vanish unlike in the pure bosonic sector. $\mathcal{L}_{11}$ yields an effective four-fermion interaction which is quartic in the fermion masses. With (8.35) and (8.36) the $\hat{D}_W^\mu\hat{V}_\mu$-terms in (7.3) and (8.27) take the form of one of the four-fermion terms already present in (8.27).

Considering renormalization and the use of the EOMs, the fermionic contribution to the Lagrangian $\mathcal{L}_\mathrm{eff}^\mathrm{ren}$(S-matrix) (7.9), which contains all effects of the heavy Higgs boson on S-matrix elements, is given by[8]

$$\begin{aligned}
\delta\mathcal{L}_\mathrm{eff}^\mathrm{ren}(\text{S-matrix}) = \frac{1}{16\pi^2}\Bigg\{ \\
&\frac{1}{32}\left(\Delta_{M_\mathrm{H}} + \frac{1}{2}\right)\frac{ig_2^2}{M_\mathrm{W}^2}\left(m_{\mathrm{f}_1}^2 - m_{\mathrm{f}_2}^2\right)\left[\bar{\hat{\Psi}}_f\hat{T}\hat{\not{D}}_{f,-}\omega_-\hat{\Psi}_f + \mathrm{h.c.}\right] \\
&-\frac{1}{16}\left(\Delta_{M_\mathrm{H}} + \frac{5}{2}\right)\frac{ig_2^2}{M_\mathrm{W}^2}\bar{\hat{\Psi}}_f\Big[2M_\mathrm{f}\hat{U}^\dagger\hat{\not{V}}\hat{U}M_\mathrm{f}\omega_+ - \left(\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\hat{\not{V}} + \hat{\not{V}}\hat{U}M_\mathrm{f}^2\hat{U}^\dagger\right)\omega_-\Big]\hat{\Psi}_f \\
&-\frac{1}{8}\left(\Delta_{M_\mathrm{H}} + \frac{21}{2} - \frac{3\sqrt{3}\pi}{2}\right)\frac{g_2^2}{M_\mathrm{W}^2}\bar{\hat{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_+ + M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\,\mathrm{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\} \\
&-\frac{1}{192}\frac{g_2^4}{M_\mathrm{W}^4}\left[\bar{\hat{\Psi}}_f\left(\hat{U}\tau_i M_\mathrm{f}\omega_+ - M_\mathrm{f}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right]\left[\bar{\hat{\Psi}}_{f'}\left(\hat{U}\tau_i M_{\mathrm{f}'}\omega_+ - M_{\mathrm{f}'}\tau_i\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right] \\
&-\frac{3}{64}\left(\Delta_{M_\mathrm{H}} + \frac{23}{3} - \sqrt{3}\pi\right)\frac{g_2^4}{M_\mathrm{W}^4}\left[\bar{\hat{\Psi}}_f\left(\hat{U}M_\mathrm{f}\omega_+ + M_\mathrm{f}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_f\right] \\
&\qquad\times\left[\bar{\hat{\Psi}}_{f'}\left(\hat{U}M_{\mathrm{f}'}\omega_+ + M_{\mathrm{f}'}\hat{U}^\dagger\omega_-\right)\hat{\Psi}_{f'}\right]\Bigg\} \\
&+\mathcal{O}(M_\mathrm{H}^{-2}).
\end{aligned} \quad (8.38)$$

9 Discussion of the result

Inspecting the bosonic part of the effective Lagrangian (7.9), we see that the first two terms contribute to vector-boson two-point (and higher) functions, the third and the fourth to

[8] Note that in the linear parametrization of the SM no $(\bar{\hat{\Psi}}_f\tau_i\hat{\Psi}_f)^2$- and $\bar{\hat{\Psi}}_f(\hat{D}_\mathrm{W}\hat{V})\hat{\Psi}_f$-terms are generated directly, because they correspond to diagrams with $\bar{\hat{\Psi}}_f\hat{\Psi}_f\varphi H$-couplings, which only exist in the non-linear parametrization. Thus, within that framework the only contribution to the $(\bar{\hat{\Psi}}_f\tau_i\hat{\Psi}_f)^2$-term comes from $\mathcal{L}_{11}$ according to (8.36). Applying (8.36) to the $\mathcal{L}_{11}$-term in Ref. [6], where the linear parametrization was used, we find that our result for the $(\bar{\hat{\Psi}}_f\tau_i\hat{\Psi}_f)^2$-term is consistent with the one of Ref. [6]; i.e. the difference in the $\mathcal{L}_{11}$-term between Ref. [6] and this article is compensated by fermionic terms.

vector-boson three-point (and higher) functions, and the last two to vector-boson four-point functions. This means that the first two terms parametrize the effects of the heavy Higgs boson on LEP 1 physics, the next two become relevant for LEP 2 physics, and the last two for LHC physics.

By naive power counting one expects that integrating out the Higgs boson generates dimension-2 terms at $\mathcal{O}(M_{\mathrm{H}}^2)$ and dimension-4 terms at $\mathcal{O}(M_{\mathrm{H}}^0)$ (i.e. proportional to $\log M_{\mathrm{H}}$) [6, 18, 19]. Actually, only those effective terms which do not violate custodial $\mathrm{SU}(2)_{\mathrm{W}}$ invariance are generated at this order. However, the effective Lagrangian (7.3) contains only one custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-violating term[9], namely $\mathcal{L}_0$ (5.1). This is a dimension-2 term; nevertheless it is only generated at $\mathcal{O}(M_{\mathrm{H}}^0)$. There are 7 custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-violating dimension-4 terms [6, 19] but none of them is generated at $\mathcal{O}(M_{\mathrm{H}}^0)$. This means that custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-breaking terms are suppressed by at least a factor of $M_{\mathrm{W}}^2/M_{\mathrm{H}}^2$ in comparison to the prediction of naive power counting. Actually, the reason for this suppression also follows from a (slightly more involved) power counting argument: The custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-breaking terms are those which explicitly contain the operator P_3 defined in (3.10). However, as shown in Sect. 4, all contributions from that operator to $\tilde{\tilde{\Delta}}_H(x, \partial_x + ip)$ and thus to $\mathcal{L}_{\mathrm{eff}}$ have the form $(M_{\mathrm{W}}^2 - M_{\mathrm{Z}}^2)P_3$ (see eq. (4.9)). Therefore, P_3 always occurs together with a power of M_{W}^2 and for dimensional reasons these contributions are suppressed by an additional power of $M_{\mathrm{W}}^2/M_{\mathrm{H}}^2$.

The fermionic part of the effective Lagrangian (8.38) contains contributions to fermion two-point functions in the first term, to fermion-fermion-vector couplings in the first and the second term, fermion-fermion-vector-vector couplings in the third term and four-fermion interactions in the last two terms. All effective fermionic couplings have at least a factor $m_{\mathrm{f}_i}/M_{\mathrm{W}}$. Consequently, the fermionic part of the effective Lagrangian (8.38) vanishes for massless fermions (and is suppressed for light fermions), i.e. the purely bosonic effective Lagrangian (7.9) describes all $\mathcal{O}(M_{\mathrm{H}}^0)$-effects of the heavy Higgs boson in this case. Unlike the bosonic terms, the effective fermionic interactions of course break custodial $\mathrm{SU}(2)_{\mathrm{W}}$ owing to the occurrence of the non-degenerate fermion-mass matrix M_{f} (8.2). Furthermore, also effective fermionic terms of dimension 5 or 6 are generated at $\mathcal{O}(M_{\mathrm{H}}^0)$ and not only dimension-4 terms like in the bosonic sector.

In analogy to the simpler SU(2) toy model considered in Ref. [1], we find that the limit $M_{\mathrm{H}} \to \infty$ of the standard model at one loop is the corresponding GNLSM plus the effective interaction terms given in (7.9) and (8.38), which describe the one-loop effects of the heavy Higgs boson. In order to calculate the complete one-loop effects to a given process at $\mathcal{O}(M_{\mathrm{H}}^0)$, one still has to consider the effects of the light quantum fields in the GNLSM Lagrangian. The coefficients of the effective terms in (7.9), (8.38) contain logarithmic divergences Δ (see (A.2)). Since the SM is is renormalizable, these UV-divergences necessarily cancel against the logarithmically divergent contributions of the non-renormalizable one-loop Lagrangian of the GNLSM $\mathcal{L}_{\mathrm{GNLSM}}^{1-\mathrm{loop}}$ in (7.6). These have been

[9]Strictly speaking, the designation "custodial $\mathrm{SU}(2)_{\mathrm{W}}$ invariance", i.e. global $\mathrm{SU}(2)_{\mathrm{W}}$ invariance in the absence of the B-field, is misleading, because locally $\mathrm{SU}(2)_{\mathrm{W}} \times \mathrm{U}(1)_{\mathrm{Y}}$-invariant terms as in (5.1) automatically fulfill this invariance. In the literature the expression "custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-invariant" is commonly used for terms which are custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-invariant when additionally the Goldstone fields are disregarded (rhs of (5.1)), and in this sense it also has to be understood in this article. The custodial-$\mathrm{SU}(2)_{\mathrm{W}}$-violating terms are then those containing the operator $\hat{T}$ (5.2) but not explicitly the $\hat{B}$-field.

calculated for the bosonic part of the GNLSM in Ref. [19] and for the dimension-4 terms of the fermionic part in Ref. [21]. Comparing our result (7.9) with Ref. [19] and the first two terms in (8.38) with Ref. [21][10] we find that the divergencies indeed cancel. In particular, since logarithmic divergences and $\log M_{\mathrm{H}}$-terms always occur in the linear combination $\Delta_{M_{\mathrm{H}}}$ (A.2), the logarithmically divergent one-loop contributions of the GNLSM to S-matrix elements coincide with the logarithmically M_{H}-dependent one-loop contributions in the SM, if one replaces

$$\frac{2}{4-D} - \gamma_E + \log(4\pi) + \log\mu^2 \quad \rightarrow \quad \log M_{\mathrm{H}}^2. \tag{9.1}$$

However, the Lagrangians (7.9) and (8.38) contain additional finite and M_{H}-independent contributions. Thus, the $\log M_{\mathrm{H}}$ one-loop contributions to the S-matrix in the SM can alternatively be calculated in the GNLSM with the replacement (9.1), however the constant contribution cannot be calculated within this model. Therefore, the GNLSM is *not* identical to the limit $M_{\mathrm{H}} \rightarrow \infty$ of the SM beyond tree-level. In this context, it should be kept in mind that these results are derived in dimensional regularization.

The non-decoupling one-loop contributions of a heavy Higgs boson to physical observables can directly be read from the effective Lagrangians (7.9) and (8.38) simply by calculating the contributions of the generated effective terms (which only contain background fields) at tree level.

10 Physical applications

In this section we illustrate the use of the constructed effective Lagrangian. We derive the heavy-Higgs effects for some vertex functions and transition amplitudes directly from our effective Lagrangian. As a consistency check, we compare the results with those of a diagrammatical calculation.

We skip the well-known heavy-Higgs effects on LEP1 observables, where the Higgs-boson dependence is merely due to vacuum-polarization effects in the gauge-boson propagators. The corresponding $\log M_{\mathrm{H}}$-terms can easily be read off from the first two lines in the effective Lagrangian (7.9).

10.1 Bosonic processes

We start by considering vector-boson scattering. In Ref. [22] the heavy-Higgs effects on the one-loop radiative corrections to $\gamma\gamma \rightarrow \mathrm{W}^+\mathrm{W}^-$ in the SM have been investigated and related to the corrections within the GNLSM. From our Lagrangian (7.9) it is very easy to reproduce the results given there so that we do not repeat the explicit formulas. We just note that no $\log M_{\mathrm{H}}$-terms in the SM with a heavy Higgs boson appear, i.e. the one-loop corrections to $\gamma\gamma \rightarrow \mathrm{W}^+\mathrm{W}^-$ in the GNLSM are UV-finite despite of the non-renormalizability of the GNLSM.

[10] In order to compare (8.38) with Ref. [21] one has to decompose M_{f} (8.2) according to (8.29). The logarithmically divergent contributions of the GNLSM to the fermionic dimension-5 and -6 terms (third to fifth term in (8.38)) have to our knowledge not been calculated in the literature.

As a second example we treat the process

$$\mathrm{W}^{+}(k_1,\lambda_1)+\mathrm{W}^{-}(k_2,\lambda_2)\rightarrow \mathrm{W}^{+}(k_3,\lambda_3)+\mathrm{W}^{-}(k_4,\lambda_4)$$

in the heavy-Higgs limit. Here $k_{1,2}$ denote the (incoming) momenta of the incoming W bosons, and $k_{3,4}$ the (outgoing) momenta of the outgoing W bosons. The corresponding Mandelstam variables are defined by

$$s=(k_1+k_2)^2,\quad t=(k_1-k_3)^2,\quad u=(k_1-k_4)^2. \tag{10.1}$$

The helicity states are labeled by λ_i, and the corresponding polarization vectors by ε_i. In the limit $s,-t,-u,M_{\mathrm{W}}^2\ll M_{\mathrm{H}}^2$ the tree-level transition amplitude $\mathcal{M}_0$ is given by

$$\begin{aligned}\mathcal{M}_0 = {}& \frac{4\pi\alpha}{s_{\mathrm{W}}^2}\left[\frac{\mathcal{M}_s}{s-M_{\mathrm{Z}}^2}+(\varepsilon_1\cdot\varepsilon_4^*)(\varepsilon_2\cdot\varepsilon_3^*)-(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3^*\cdot\varepsilon_4^*)\right]-4\pi\alpha\frac{M_{\mathrm{Z}}^2}{s(s-M_{\mathrm{Z}}^2)}\mathcal{M}_s\\ &+\ \text{crossed}\ +\ \mathcal{O}(M_{\mathrm{H}}^{-2}),\end{aligned}\tag{10.2}$$

where crossing means the interchanges $\varepsilon_2\leftrightarrow\varepsilon_3^*, k_2\leftrightarrow -k_3$. Note that the single contributions in (10.2) are arranged according to the independent couplings $g_2=e/s_{\mathrm{W}}$ and e, where $\alpha=e^2/4\pi$ is the usual fine-structure constant. The following shorthands have been introduced,

$$\begin{aligned}\mathcal{M}_s = {}& \mathcal{M}_s'+(u-t)(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3^*\cdot\varepsilon_4^*)\\ &+2(\varepsilon_1\cdot\varepsilon_2)\left[(k_1\cdot\varepsilon_4^*)(k_2\cdot\varepsilon_3^*)-(k_1\cdot\varepsilon_3^*)(k_2\cdot\varepsilon_4^*)\right]\\ &+2(\varepsilon_3^*\cdot\varepsilon_4^*)\left[(k_3\cdot\varepsilon_2)(k_4\cdot\varepsilon_1)-(k_3\cdot\varepsilon_1)(k_4\cdot\varepsilon_2)\right],\\ \mathcal{M}_s' = {}& \quad 4(k_1\cdot\varepsilon_2)\left[(k_3\cdot\varepsilon_4^*)(\varepsilon_1\cdot\varepsilon_3^*)-(k_4\cdot\varepsilon_3^*)(\varepsilon_1\cdot\varepsilon_4^*)\right]\\ &+4(k_2\cdot\varepsilon_1)\left[(k_4\cdot\varepsilon_3^*)(\varepsilon_2\cdot\varepsilon_4^*)-(k_3\cdot\varepsilon_4^*)(\varepsilon_2\cdot\varepsilon_3^*)\right]\\ &+2(\varepsilon_1\cdot\varepsilon_2)\left[(k_1\cdot\varepsilon_4^*)(k_2\cdot\varepsilon_3^*)-(k_1\cdot\varepsilon_3^*)(k_2\cdot\varepsilon_4^*)\right]\\ &+2(\varepsilon_3^*\cdot\varepsilon_4^*)\left[(k_3\cdot\varepsilon_2)(k_4\cdot\varepsilon_1)-(k_3\cdot\varepsilon_1)(k_4\cdot\varepsilon_2)\right].\end{aligned}\tag{10.3}$$

Now, we consider the one-loop effects of the heavy Higgs boson to this process, which can be obtained from the effective Lagrangians (7.3) or (7.9), respectively, simply by calculating the tree-level contributions of $\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}}$. As explained above, only the terms in (7.9) are relevant for the contribution to the S-matrix element, whereas the additional terms in (7.3) cancel exactly. The effective Lagrangian yields the difference $\delta\mathcal{M}=\delta\mathcal{M}_{\mathrm{SM}}-\delta\mathcal{M}_{\mathrm{GNLSM}}$ (in dimensional regularization) between the one-loop corrections to the amplitude in the SM with a heavy Higgs boson and the GNLSM, respectively. One finds

$$\begin{aligned}\delta\mathcal{M} = {}& \frac{\alpha^2}{s_{\mathrm{W}}^4}\Bigg[-\frac{5}{6}\left(\Delta_{M_{\mathrm{H}}}+\frac{19}{30}\right)\left(\frac{\mathcal{M}_s}{s-M_{\mathrm{Z}}^2}+(\varepsilon_1\cdot\varepsilon_4^*)(\varepsilon_2\cdot\varepsilon_3^*)-(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3^*\cdot\varepsilon_4^*)\right)\\ &\quad -\frac{1}{12}\left(\Delta_{M_{\mathrm{H}}}+\frac{17}{6}\right)(\varepsilon_1\cdot\varepsilon_4^*)(\varepsilon_2\cdot\varepsilon_3^*)\\ &\quad -\frac{1}{6}\left(\Delta_{M_{\mathrm{H}}}+\frac{175}{12}-\frac{9\sqrt{3}\pi}{4}\right)(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3^*\cdot\varepsilon_4^*)\Bigg]\\ &-\frac{\alpha^2}{s_{\mathrm{W}}^2}\frac{1}{6}\frac{M_{\mathrm{Z}}^2}{s(s-M_{\mathrm{Z}}^2)}\mathcal{M}_s'\quad +\ \text{crossed}\ +\ \mathcal{O}(M_{\mathrm{H}}^{-2}).\end{aligned}\tag{10.4}$$

The single terms in (10.4) are arranged such that only the second and the third line yield contributions of order xy/M_{W}^4 $(x, y = s, t, u)$ in the high-energy limit for purely longitudinally polarized W bosons. These terms entirely originate from the genuine four-point operators in the effective Lagrangian, i.e. from $\mathcal{L}_4$ and $\mathcal{L}_5$. The complete xy/M_{W}^4-terms of the one-loop correction to $\mathrm{W}_{\mathrm{L}}^+\mathrm{W}_{\mathrm{L}}^- \to \mathrm{W}_{\mathrm{L}}^+\mathrm{W}_{\mathrm{L}}^-$ in the limit $M_{\mathrm{W}}^2 \ll s, -t, -u \ll M_{\mathrm{H}}^2$ were calculated in Ref. [23] and Ref. [24] in an SU(2) gauge theory and the SM, respectively. Comparing our results with the ones given there, we find agreement for the $\log M_{\mathrm{H}}$-terms [11] and the "$\sqrt{3}\pi$" term, which stems from Higgs-mass renormalization. The remaining M_{H}-independent xy/M_{W}^4-terms are of course different since additional terms of this kind originate from bosonic loops without Higgs bosons, which are equal in the SM and GNLSM. As a consistency check, we have also calculated $\delta\mathcal{M}$ diagrammatically and found the same result. Figures 2,3,4 show the Higgs-mass-dependent subdiagrams contributing in $\mathcal{O}(M_{\mathrm{H}}^0)$ to Feynman diagrams and counterterms which are reducible with respect to light particles. The irreducible $\mathcal{O}(M_{\mathrm{H}}^0)$ contributions and those which are reducible with respect to the heavy Higgs field (which correspond to the irreducible contributions of $\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}}$) are depicted in Fig. 5 (where all fields are assumed to be incoming). The advantage of our effective-Lagrangian approach is obvious: in a diagrammatical calculation all these diagrams have to be evaluated while in the effective-Lagrangian calculation one only has to consider the tree-level contributions of $\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}}$ (7.9).

10.2 Fermionic processes

Now, we turn to examples involving massive fermions. The only Higgs-mass-dependent contributions of the effective Lagrangian (8.26) to the fermion self-energy are contained in the first two terms, viz.

$$\begin{aligned}
\delta\Sigma_{\mathrm{L}}^{\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i}(k^2) = \delta\Sigma_{\mathrm{R}}^{\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i}(k^2) &= \frac{g_2^2}{64\pi^2}\frac{m_{\mathrm{f}_i}^2}{M_{\mathrm{W}}^2}\left(I_{011} - 2I_{112}\right) \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}), \\
\delta\Sigma_{\mathrm{S}}^{\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i}(k^2) &= \frac{g_2^2}{64\pi^2}\frac{m_{\mathrm{f}_i}^2}{M_{\mathrm{W}}^2} I_{011} \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}),
\end{aligned} \tag{10.5}$$

where our conventions for the fermionic self-energy follow the ones of Ref. [11]. In a diagrammatical calculation, these contributions stem from the graph of Fig. 6.a). Using (10.5), we get for the contributions to the renormalization constants,

$$\begin{aligned}
\left.\frac{\delta m_{\mathrm{f}_i}}{m_{\mathrm{f}_i}}\right|_{\mathrm{H}} &= \frac{g_2^2}{32\pi^2}\frac{m_{\mathrm{f}_i}^2}{M_{\mathrm{W}}^2}\left(I_{011} - 2I_{112}\right) \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}), \\
\left.\delta Z_{f_i}^{\sigma}\right|_{\mathrm{H}} &= -\frac{g_2^2}{64\pi^2}\frac{m_{\mathrm{f}_i}^2}{M_{\mathrm{W}}^2}\left(I_{011} - 2I_{112}\right) \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}).
\end{aligned} \tag{10.6}$$

The field-renormalization constants $\delta Z_{f_i}^{\sigma}$ are chosen such that the residue of the f_i propagator equals one. Combining (10.5) and (10.6), we obtain that the renormalized fermion self-energy contains no Higgs-mass-dependent terms of $\mathcal{O}(M_{\mathrm{H}}^0)$,

$$\delta\Sigma_{\mathrm{L/R/S}}^{\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i,\mathrm{ren}}(k^2) \;=\; \mathcal{O}(M_{\mathrm{H}}^{-2}). \tag{10.7}$$

[11]The terms of the order $(xy/M_{\mathrm{W}}^4)\log M_{\mathrm{H}}$ were already given in Ref. [5] by calculating the logarithmic divergences (Δ-terms) within the GNLSM and using the replacement (9.1).

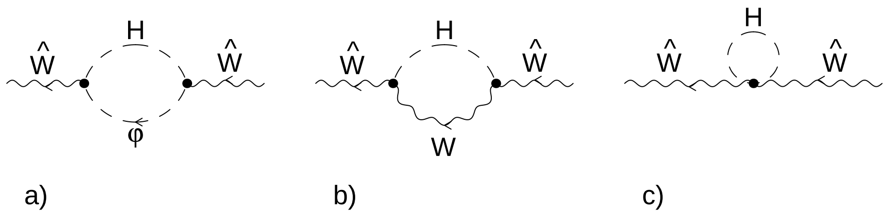


Figure 2: Higgs diagrams to the $\hat{W}$–self-energy.

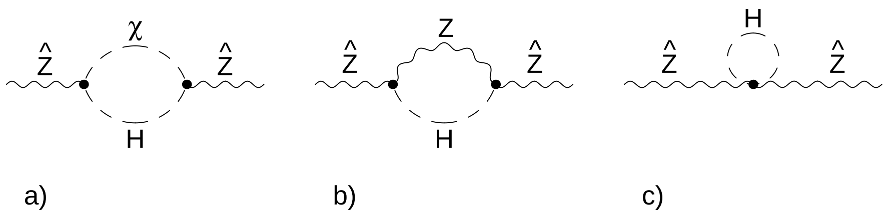


Figure 3: Higgs diagrams to the $\hat{Z}$–self-energy.

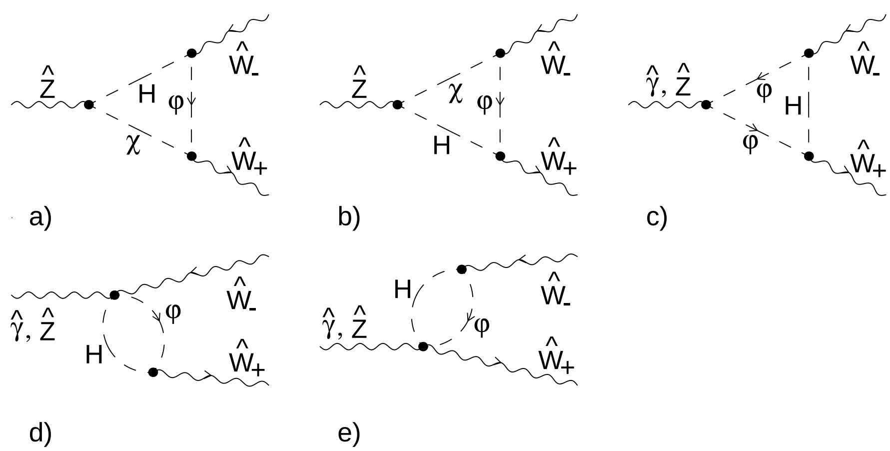


Figure 4: Higgs diagrams of $\mathcal{O}(M_{\mathrm{H}}^0)$ for the $\hat{Z}\hat{W}^+\hat{W}^-$- and $\hat{A}\hat{W}^+\hat{W}^-$-vertex functions.

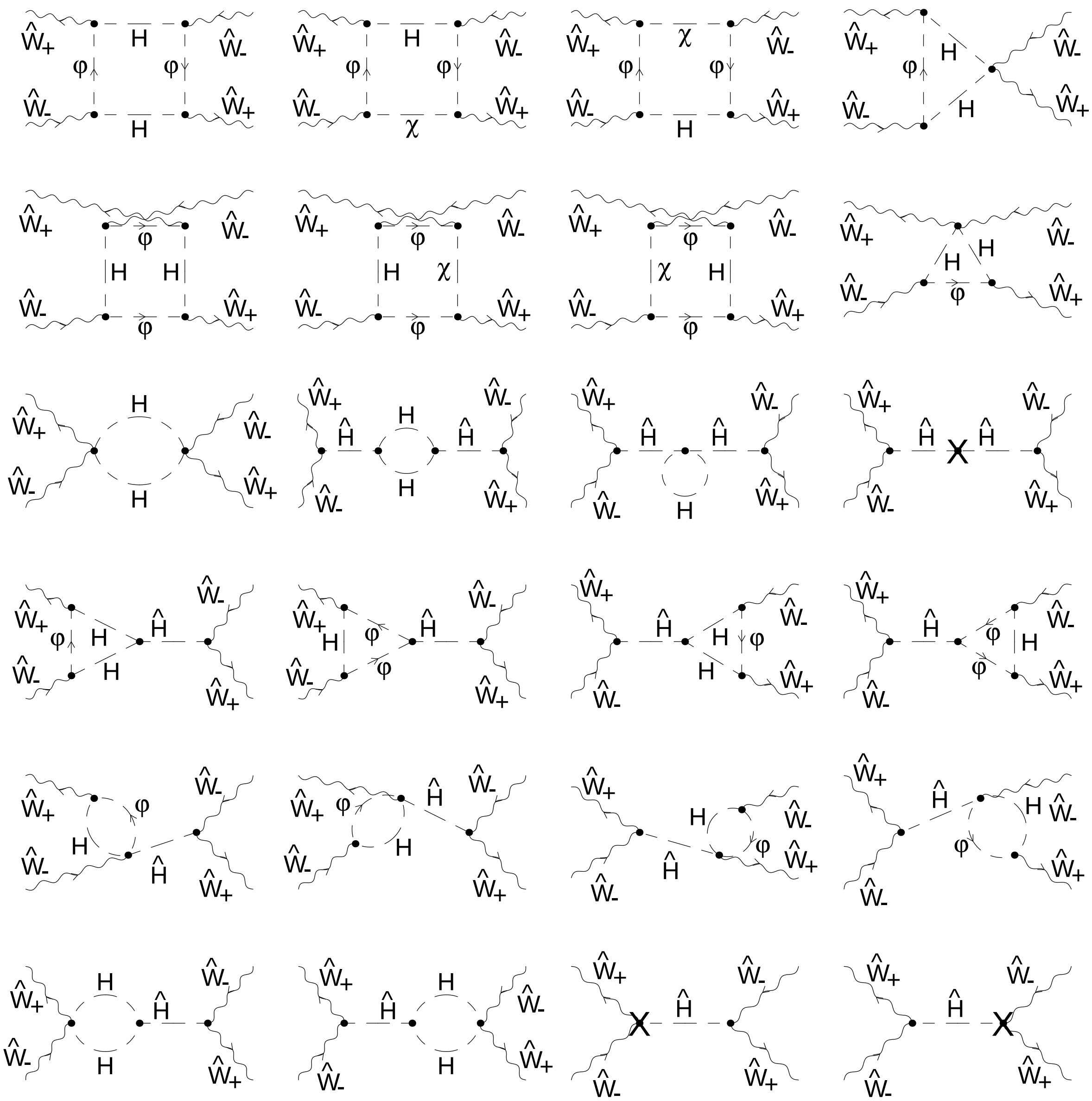
The Higgs-mass dependence of the photon-fermion-fermion vertex is contained in the second term in (8.26), which yields

$$\delta\Gamma_\mu^{\hat{A}\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i}(k,\bar{p},p) = -\frac{iQ_{\mathrm{f}_i}eg_2^2}{64\pi^2}\frac{m_{\mathrm{f}_i}^2}{M_{\mathrm{W}}^2}\gamma_\mu\left(I_{011}-2I_{112}\right) \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}). \tag{10.8}$$

In a diagrammatical calculation one has to calculate the graph shown in Fig. 6.b). Again after renormalization no $\mathcal{O}(M_{\mathrm{H}}^0)$ survives for this vertex function,

$$\delta\Gamma_\mu^{\hat{A}\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i,\mathrm{ren}}(k,\bar{p},p) \;=\; \mathcal{O}(M_{\mathrm{H}}^{-2}). \tag{10.9}$$

The $\mathcal{O}(M_{\mathrm{H}}^0)$ contributions to $\delta\Gamma_\mu^{\hat{A}\hat{\bar{\mathrm{f}}}_i\hat{\mathrm{f}}_i}$ are cancelled by the fermionic wave-function corrections, and the charge renormalization constant does not contain terms of $\mathcal{O}(M_{\mathrm{H}}^0)$. From



+ crossed graphs (external $\hat{W}^-$ interchanged)

Figure 5: Higgs diagrams of $\mathcal{O}(M_{\mathrm{H}}^0)$ for the one-particle-irreducible and the heavy-Higgs reducible $\hat{W}^+\hat{W}^-\hat{W}^+\hat{W}^-$–four-point function.

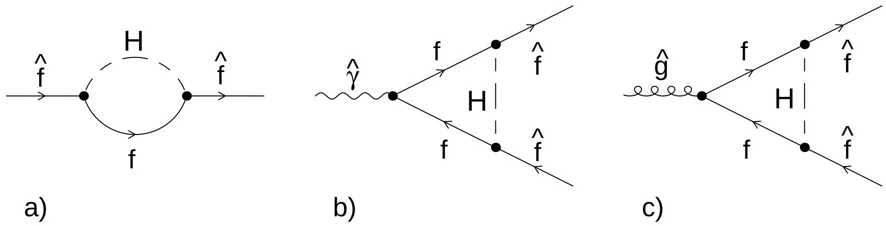


Figure 6: Higgs diagrams contributing to the a) fermion self-energy, b) photon-fermion-fermion vertex, c) gluon-fermion-fermion vertex.

(10.7) and (10.9) we draw the conclusion that no $\mathcal{O}(M_{\mathrm{H}}^0)$-terms of the effective Lagrangian contribute e.g. to the SM one-loop corrections to $\gamma\gamma \to \mathrm{f}_i\bar{\mathrm{f}}_i$. This means that the SM one-loop prediction for $\gamma\gamma \to \mathrm{f}_i\bar{\mathrm{f}}_i$ in the heavy-Higgs limit approaches asymptotically the GNLSM correction, which is UV-finite either. The analogue conclusion also holds for gluon-gluon fusion, $\mathrm{gg} \to \mathrm{f}_i\bar{\mathrm{f}}_i$, since the Higgs-mass-dependent subdiagrams of $\mathcal{O}(M_{\mathrm{H}}^0)$ are the same as for $\gamma\gamma \to \mathrm{f}_i\bar{\mathrm{f}}_i$ with the external photons replaced by gluons. More precisely, only the diagrams shown in Figs. 6a),c) are relevant. For instance, the complete SM one-loop correction to $\mathrm{gg} \to \mathrm{t}\bar{\mathrm{t}}$ can be found in Ref. [25]. From the results given there, one can see that the relative one-loop correction approaches a constant for $M_{\mathrm{H}} \to \infty$ in consistence with our result.

The result (10.9) is in agreement with the one obtained in Ref. [26] for the γtt-vertex. Inspecting our corresponding results for the fermion-mass-dependent terms of the ttZ- and the tbW-vertices,

$$\begin{aligned}
\delta\Gamma_\mu^{\hat{\mathrm{Z}}\hat{\bar{\mathrm{t}}}\hat{\mathrm{t}},\mathrm{ren}}(k,\bar{p},p)\Big|_{M_{\mathrm{f}}} &= \frac{ig_2^3}{128\pi^2 c_{\mathrm{W}}}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\gamma_\mu\gamma_5\,(I_{011}-6I_{112}) + (k_\mu\text{-terms}) + \mathcal{O}(M_{\mathrm{H}}^{-2}),\\
\delta\Gamma_\mu^{\hat{\mathrm{W}}\hat{\bar{\mathrm{t}}}\hat{\mathrm{b}},\mathrm{ren}}(k,\bar{p},p)\Big|_{M_{\mathrm{f}}} &= -\frac{ig_2^3}{128\sqrt{2}\pi^2}\gamma_\mu\left(\frac{m_{\mathrm{t}}^2+m_{\mathrm{b}}^2}{M_{\mathrm{W}}^2}\omega_- - 2\frac{m_{\mathrm{t}}m_{\mathrm{b}}}{M_{\mathrm{W}}^2}\omega_+\right)(I_{011}-6I_{112})\\
&\quad + (k_\mu\text{-terms}) + \mathcal{O}(M_{\mathrm{H}}^{-2}).
\end{aligned} \tag{10.10}$$

which are contained in the second and third terms in (8.26)[12], we also find agreement with Ref. [26], where the $m_{\mathrm{t}}^2 \log M_{\mathrm{H}}$-terms were calculated.

Finally, we investigate the heavy-Higgs effects to the top-quark decay $\mathrm{t} \to \mathrm{W}^+\mathrm{b}$. In lowest order the transition amplitude for this process is given by

$$\mathcal{M}_0 = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\bar{u}(p_{\mathrm{b}})\not{\varepsilon}_{\mathrm{W}}^*\omega_- u(p_{\mathrm{t}}), \tag{10.11}$$

with p_{t} and $u(p_{\mathrm{t}})$ (p_{b} and $u(p_{\mathrm{b}})$) denoting the incoming (outgoing) momentum and spinor for the top(bottom)-quark, respectively. ε_{W} represents the polarization vector of the W

[12] As indicated in (10.10), there are also k_μ-terms stemming from the fifth term in (8.26). As explained in Subsect. 8.7, this term becomes a four-fermion term in $\mathcal{L}_{\mathrm{eff}}^{\mathrm{ren}}$(S-matrix) (8.38) after applying the EOM. Thus, its contribution is not considered here.

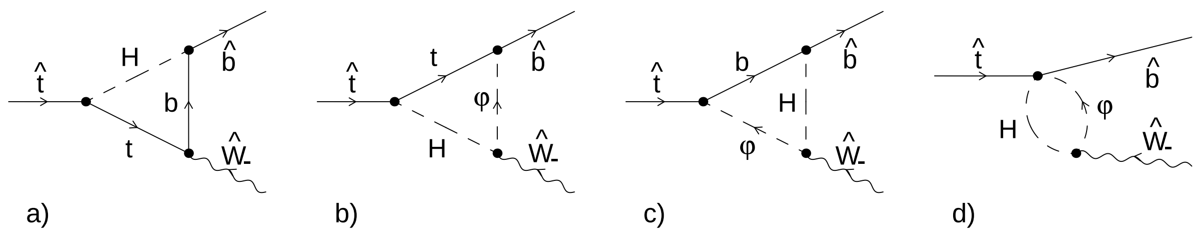


Figure 7: Higgs diagrams of $\mathcal{O}(M_{\mathrm{H}}^0)$ for the $\hat{t}\hat{\bar{b}}\hat{W}^-$-vertex function.

boson. The complete difference $\delta\mathcal{M} = \delta\mathcal{M}_{\mathrm{SM}} - \delta\mathcal{M}_{\mathrm{GNLSM}}$ can easily be calculated from the effective interaction terms (10.10). We obtain

$$\delta\mathcal{M} = \frac{e\alpha}{16\sqrt{2}\pi s_{\mathrm{W}}^3}\Bigg\{ \quad \bar{u}(p_{\mathrm{b}})\not{\varepsilon}_{\mathrm{W}}^*\omega_- u(p_{\mathrm{t}}) \left[\left(\frac{m_{\mathrm{t}}^2+m_{\mathrm{b}}^2}{M_{\mathrm{W}}^2}\right)\frac{1}{4}\left(\Delta_{M_{\mathrm{H}}}+\frac{5}{2}\right) - \frac{11}{6}\left(\Delta_{M_{\mathrm{H}}}+\frac{5}{6}\right)\right]$$
$$- \bar{u}(p_{\mathrm{b}})\not{\varepsilon}_{\mathrm{W}}^*\omega_+ u(p_{\mathrm{t}})\frac{m_{\mathrm{t}}m_{\mathrm{b}}}{M_{\mathrm{W}}^2}\frac{1}{2}\left(\Delta_{M_{\mathrm{H}}}+\frac{5}{2}\right)\Bigg\} \;+\; \mathcal{O}(M_{\mathrm{H}}^{-2}). \qquad (10.12)$$

Alternatively, (10.12) could be derived by calculating the diagrams shown in Fig. 7, where graph 7.d) does not contribute to the S-matrix element. The term in (10.12) which is not multiplied by fermion masses is entirely due to coupling-constant and W-wave-function renormalization. It is associated with the well-known variable Δr, i.e. it is absent in a renormalization scheme, where the Fermi constant G_F is used as an input parameter instead of the W mass M_{W}. The M_{H} dependence of the top width originating from the remaining $\log M_{\mathrm{H}}$-terms in (10.12) is e.g. numerically discussed in Ref. [27], where the complete one-loop SM correction is calculated. The $(m_{\mathrm{t}}^2/M_{\mathrm{W}}^2)\log M_{\mathrm{H}}$-term can for instance be found in Ref. [28] in agreement with our result.

11 Conclusion

In this article we have integrated out the Higgs boson in the electroweak standard model directly in the path integral, assuming that it is very heavy. We have expressed all non-decoupling effects, i.e. effects of $\mathcal{O}(M_{\mathrm{H}}^0)$, of the heavy Higgs boson (including fermionic effects) in terms of an effective Lagrangian, from which the leading contributions of the Higgs boson to physical parameters and scattering processes can easily be read.

For the bosonic sector of the SM, this result itself is essentially already known from the diagrammatical calculation of Ref. [6]. However, we have derived it in a completely different way, viz. by integrating out the Higgs boson directly in the path integral instead of calculating Feynman diagrams and matching the full theory to the effective one. The functional method is a methodical progress for several reasons: As pointed out in Ref. [16], diagrammatical calculations like those in Ref. [6] cannot determine the full content of Green function but only the "physically relevant parts". This is due to problems with gauge invariance of the matching conditions. However, owing to the application of the background-field method and the Stueckelberg formalism, our direct calculation yields

the complete effective Lagrangian in a manifestly gauge-invariant form without those problems. Moreover, the functional method is a huge technical simplification in comparison to the diagrammatical one, because in the functional approach the effective Lagrangian – which contains contributions to many Green functions – is generated *directly* by integrating out the heavy field. In a diagrammatical calculation one has to calculate various Green functions (i.e. very many Feynman graphs), to write down all effective interaction terms which could possibly be generated, and then determine the effective Lagrangian by comparing coefficients [6]. We can use the convenient matrix notation throughout, i.e. we do not have to specify the single components of the fields. For the background fields we even do not have to introduce the physical basis. A striking simplification within our method is the fact that it is completely obvious that only 7 of 14 possible effective bosonic interaction terms of dimension 4 (or 2) are generated in $\mathcal{O}(M_{\mathrm{H}}^0)$ at one loop, i.e. that the 7 custodial-SU(2)$_{\mathrm{W}}$-violating dimension-4 terms are only of $\mathcal{O}(M_{\mathrm{H}}^{-2})$. This result was also found by the diagrammatical calculation in Ref. [6], however no obvious reason why these terms cancel can be seen there. In our direct calculation these terms are not generated from the beginning; i.e. there are no cancellations. The suppression of all custodial-SU(2)$_{\mathrm{W}}$-violating terms by one power of $M_{\mathrm{W}}^2/M_{\mathrm{H}}^2$ follows in our approach from a simple power-counting argument.

In addition, we also considered the fermionic sector of the standard model when integrating out the Higgs field, and constructed the fermionic terms of the effective Lagrangian. These have not been completely calculated before, neither functionally nor diagrammatically. Also this calculation becomes straightforward owing to the use of our functional method. If one applied the diagrammatical method, one would have to write down all possible effective interaction terms in order to find the matching conditions. Since even dimension-5 and -6 terms are are generated, this would be a large number, while in a functional calculation also these terms are generated directly.

In the present article we have integrated out a *non-decoupling* heavy field. However, the generalization of our method to the case of *decoupling* fields is straightforward yielding a wide field of phenomenologically interesting applications.

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Appendix

A Explicit expressions for the one-loop integrals

In Sect. 4 the construction of the unrenormalized effective Lagrangian (4.10) was traced back to the vacuum integrals $I^i_{klm}(\xi)$ defined in (4.4). Such vacuum integrals are easily calculated and their explicit expressions are already given in the appendix of Ref. [1] using dimensional regularization. The relevant $\mathcal{O}(M_{\mathrm{H}}^0)$ parts of the I^i_{klm} for $D \to 4$ are

$$I_{010} = M_{\mathrm{H}}^2(\Delta_{M_{\mathrm{H}}} + 1),$$

$$
\begin{aligned}
I^i_{011}(\xi) &= \Delta_{M_\mathrm{H}} + 1 + \mathcal{O}(M_\mathrm{H}^{-2}), \\
I_{020} &= \Delta_{M_\mathrm{H}}, \\
I^i_{111}(\xi) &= \frac{1}{4}(M_\mathrm{H}^2 + \xi M_i^2)\left(\Delta_{M_\mathrm{H}} + \frac{3}{2}\right) + \mathcal{O}(M_\mathrm{H}^{-2}), \\
I^i_{112}(\xi) &= \frac{1}{4}\left(\Delta_{M_\mathrm{H}} + \frac{3}{2}\right) + \mathcal{O}(M_\mathrm{H}^{-2}), \\
I^i_{121}(\xi) &= \frac{1}{4}\left(\Delta_{M_\mathrm{H}} + \frac{1}{2}\right) + \mathcal{O}(M_\mathrm{H}^{-2}), \\
I^i_{213}(\xi) &= \frac{1}{24}\left(\Delta_{M_\mathrm{H}} + \frac{11}{6}\right) + \mathcal{O}(M_\mathrm{H}^{-2}), \\
I^i_{222}(\xi) &= \frac{1}{24}\left(\Delta_{M_\mathrm{H}} + \frac{5}{6}\right) + \mathcal{O}(M_\mathrm{H}^{-2})
\end{aligned} \tag{A.1}
$$

with

$$
\Delta_{M_\mathrm{H}} = \Delta - \log\left(\frac{M_\mathrm{H}^2}{\mu^2}\right), \qquad \Delta = \frac{2}{4-D} - \gamma_E + \log(4\pi), \tag{A.2}
$$

and γ_E being Euler's constant. In the main part of this article we drop the index i and the argument ξ for all logarithmically divergent integrals, because these are independent of M_i^2 and ξ at $\mathcal{O}(M_\mathrm{H}^0)$.

In Sect. 6 we expressed the renormalization constant δM_H^2 (6.4) in terms of the I_{klm} and scalar two-point functions $B_0(k^2, M_1, M_2)$ defined in (6.5). The explicit expressions for the relevant B_0-functions can for instance be deduced from the general result presented in Ref. [17], leading to

$$
\begin{aligned}
B_0(M_\mathrm{H}^2, M_\mathrm{H}, M_\mathrm{H}) &= \Delta_{M_\mathrm{H}} + 2 - \frac{\pi}{\sqrt{3}}, \\
B_0(M_\mathrm{H}^2, 0, 0) &= \Delta_{M_\mathrm{H}} + 2 + i\pi.
\end{aligned} \tag{A.3}
$$

B Proof of equation (8.37)

In this appendix we prove relation (8.37), which has been used in order to simplify the $\hat{D}^\mu_W \hat{V}_\mu$-terms in $\mathcal{L}_\mathrm{eff}^\mathrm{ren}$ by using the EOMs.

First, we derive the identity

$$
P\left(UAU^\dagger\right) = U(PA)U^\dagger, \tag{B.1}
$$

where P is the projection operator (3.10), A an arbitrary 2×2-matrix and U an SU(2) matrix. Using the definition of P we find

$$
P\left(UAU^\dagger\right) = \frac{1}{2}\tau_i \,\mathrm{tr}\left\{\tau_i UAU^\dagger\right\} = \frac{1}{2}\tau_i \,\mathrm{tr}\left\{U^\dagger \tau_i U A\right\}. \tag{B.2}
$$

Owing to $\mathrm{tr}\left\{U^\dagger \tau_i U\right\} = \mathrm{tr}\left\{\tau_i\right\} = 0$, the hermitian 2×2-matrix $U^\dagger \tau_i U$ is a linear combination of Pauli matrices, i.e. it can be written as

$$
U^\dagger \tau_i U = X_{ij}\tau_j \qquad \text{with} \qquad X_{ij} = \frac{1}{2}\mathrm{tr}\left\{\tau_j U^\dagger \tau_i U\right\}. \tag{B.3}
$$

This implies

$$U\tau_j U^\dagger = \tau_i X_{ij}. \tag{B.4}$$

With (B.2), (B.3), (B.4) and (3.10) we find

$$P\left(UAU^\dagger\right) = \frac{1}{2}\tau_i X_{ij}\,\mathrm{tr}\,\{\tau_j A\} = \frac{1}{2}U\tau_j U^\dagger\,\mathrm{tr}\,\{\tau_j A\} = U(PA)U^\dagger, \tag{B.5}$$

which proves (B.1). With (B.1) one can easily derive (8.37):

$$\mathrm{tr}\,\{(PAU)(PBU)\} = \mathrm{tr}\left\{U(PAU)U^\dagger U(PBU)U^\dagger\right\} = \mathrm{tr}\,\{(PUA)(PUB)\}\,. \tag{B.6}$$

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